Principled Programming

Introduction to Coding in Any Imperative Language

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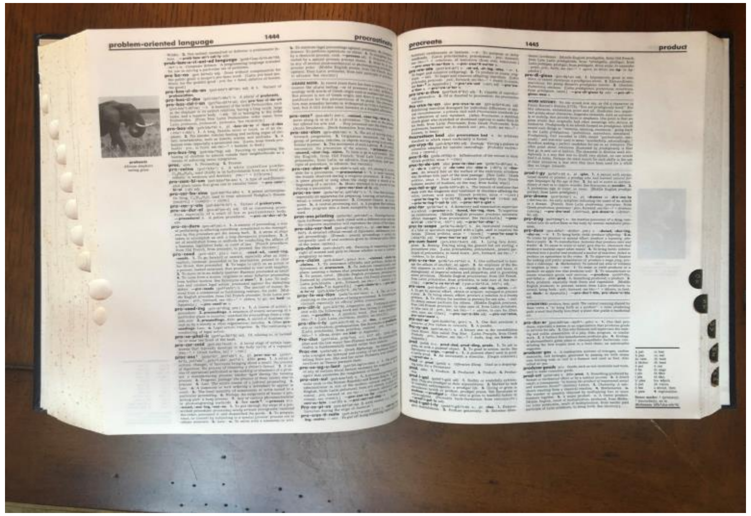
Binary Search

If you want to find the definition of the word **proboscis** in a 512-page dictionary, you wouldn't use Sequential Search starting on the first page, say, with **aardvark**. Rather, you would start roughly in the middle. From there, you would:

- Repeatedly halve the portion of the dictionary that remains under consideration, doing so by looking at the middle page of the region in hand, and discarding whichever half is revealed thereby to not contain **proboscis**.
- Once the search has been narrowed to a single page, you would look on that page to see if **proboscis** is there.
- If it is, you found its definition; otherwise, it isn't in the dictionary.



The method is called Binary Search, and is an example of a Divide and Conquer algorithm. Binary Search is astoundingly fast.

Application: Search for a value v in an unordered array `A[0..n-1]`.

```
/* Given array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
```

☞ **A statement-comment says exactly what code must accomplish, not how it does so.**

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   int k = 0;
   while ( A[k]!=v && k<n ) k++;
```

☞ **Master stylized code patterns, and use them.**

Sequential search works, but ignores the order. We can do better.

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   __________
   while ( ______ ) ___________
   __________
```

Coding order
(1) body
(2) termination
(3) initialization
(4) finalization
(5) boundary conditions

☞ **If you "smell a loop", write it down.**

Binary Search

POST

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   __________
   while ( ______ ) ___________
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```

☞ **Invent (or learn) diagrammatic ways to express concepts.**

INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   __________
   while ( ______ ) ___________
   __________
```

☞ **To get to POST iteratively, choose a weakened POST as INVARIANT.**

INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   int L = ______; int R = ______;
   while ( ______ ) __________
   __________
```

☞ **Introduce program variables whose values describe "state".**

Binary Search

VARIANT: R-L

INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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   int L = ______; int R = ______;
   while ( ______ )
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   __________
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

VARIANT: R-L
INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

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/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   int L = ______; int R = ______;
   while ( ______ )
      if ( ________ )
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/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

VARIANT: R-L

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Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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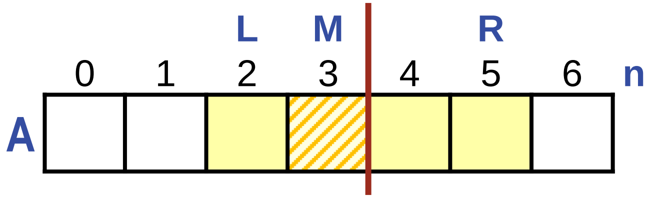
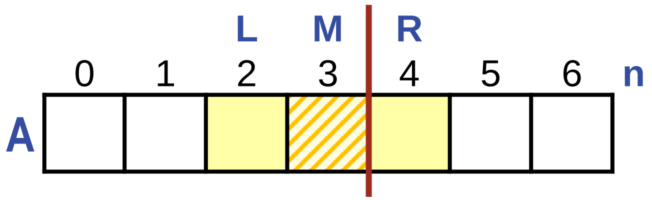
```

If you object to A[L..R] straddling the midpoint of A[0..n-1], understand that in "schematic diagrams", the exact locations of boundaries are immaterial.





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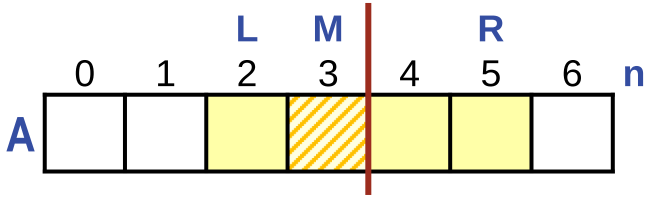
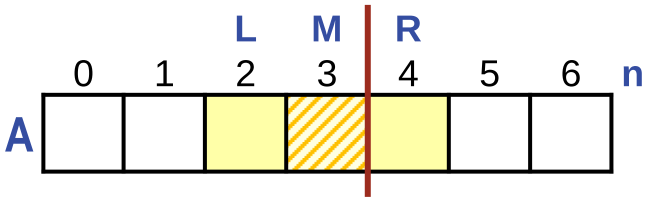
☞ **Be alert to high-risk coding steps associated with binary choices.**

Recognize that regions of even and odd lengths may need **distinct** treatments.





M is index of rightmost element of left "half".

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```

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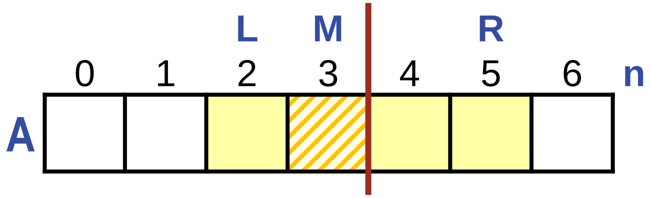
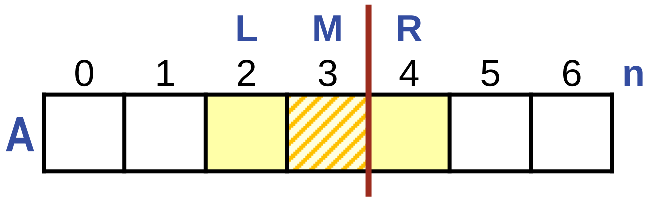
Recognize that regions of even and odd lengths may need **distinct** treatments, but hope for a **uniform** treatment.





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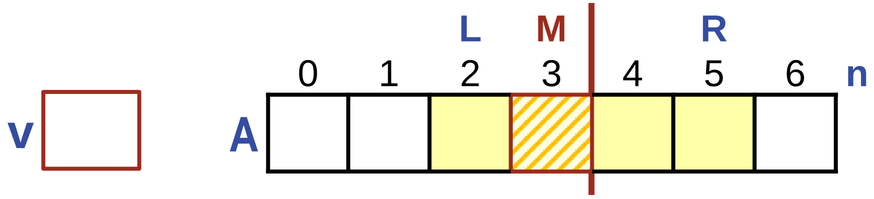

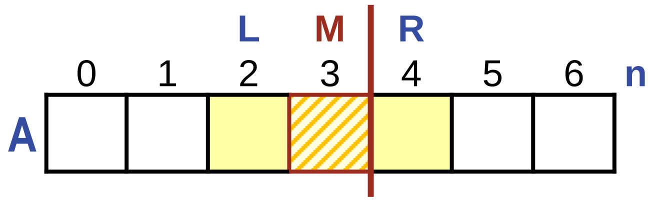

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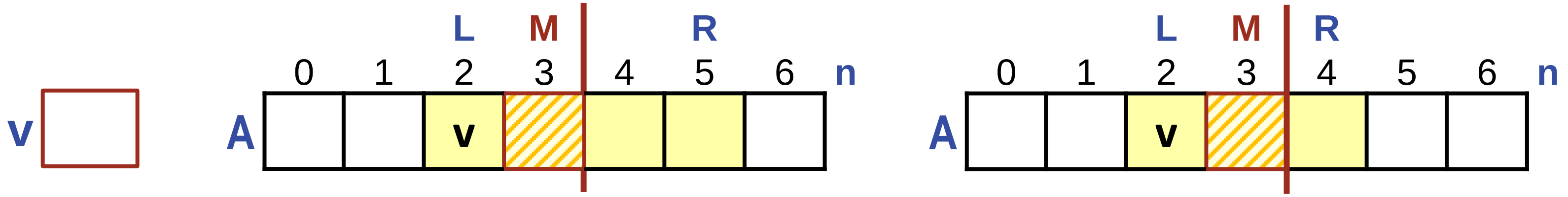

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   while ( ______ )
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   __________
```

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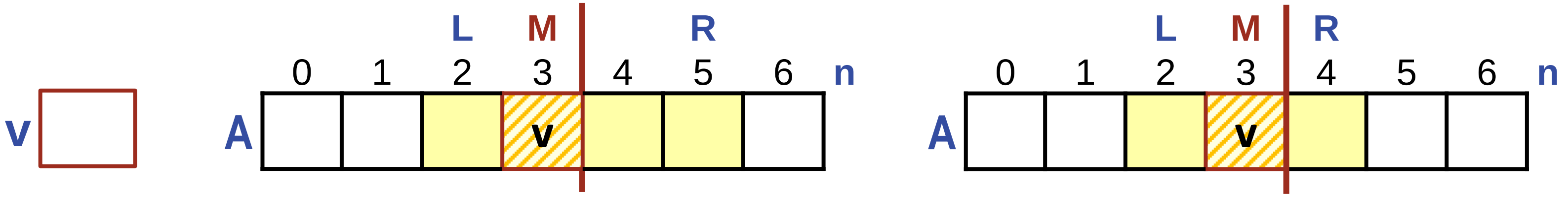

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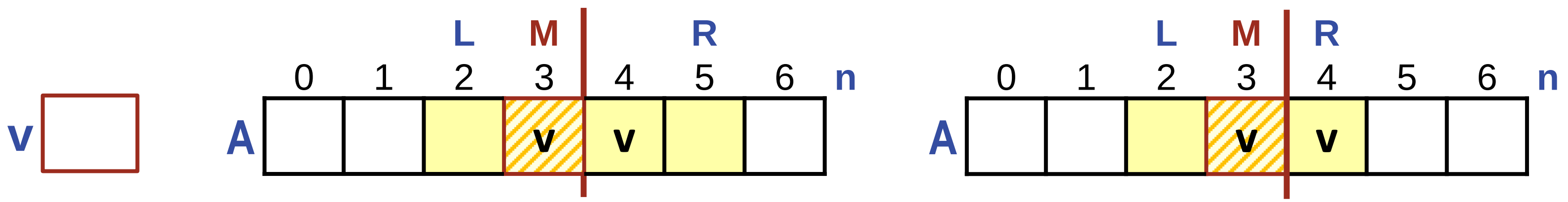

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   __________
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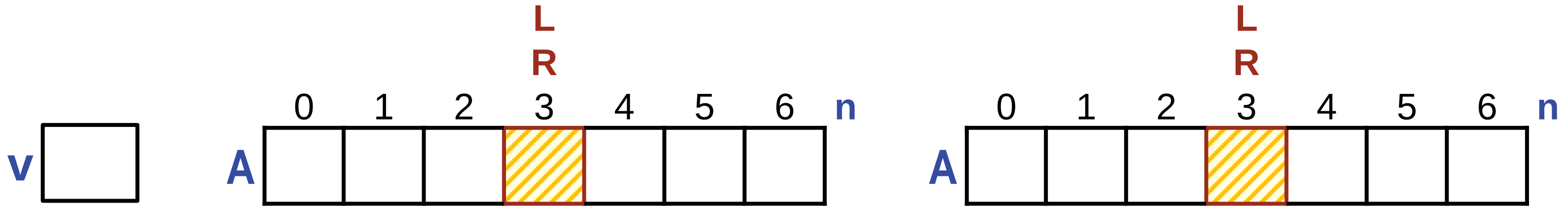
☞ **Be alert to high-risk coding steps associated with binary choices.**

Duplicate instances of v in `A[L..R]` may escape, but not all of them.



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   while ( L != R )
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   __________
```

Coding order
(1) body
(2) termination
(3) initialization
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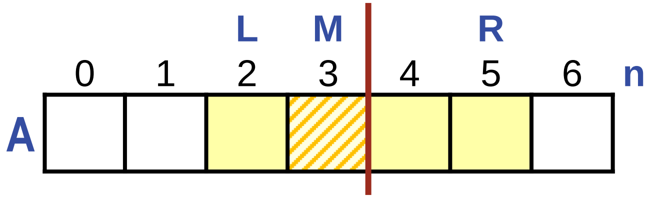
(5) boundary conditions



VARIANT: R-L 4 ➔ 2 or 2

Application: Search for a value v in an ordered array A[0..n-1].

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      else L = M+1;           // Select right "half".
   __________
```

Coding order
(1) body
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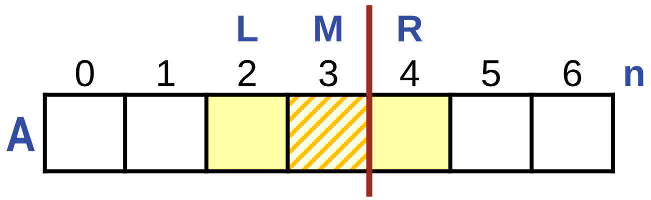
Confirm that the **VARIANT** decreases on every iteration.



VARIANT: R-L 3 ➔ 2 or 1

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if no v in A. */
   int L = ______; int R = ______;
   M = (L+R)/2;              // Compute “midpoint”.
   while ( L != R )
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   __________
```

Coding order
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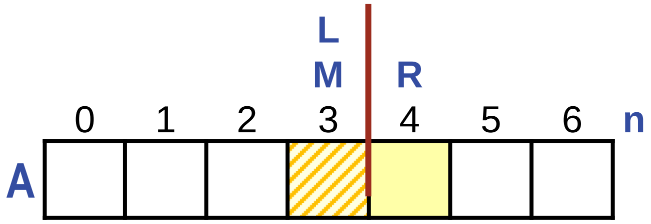

VARIANT: R-L 2 ➔ 1 or 1

Application: Search for a value v in an ordered array `A[0..n-1]`.

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   while ( L != R )
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   __________
```

Coding order
(1) body
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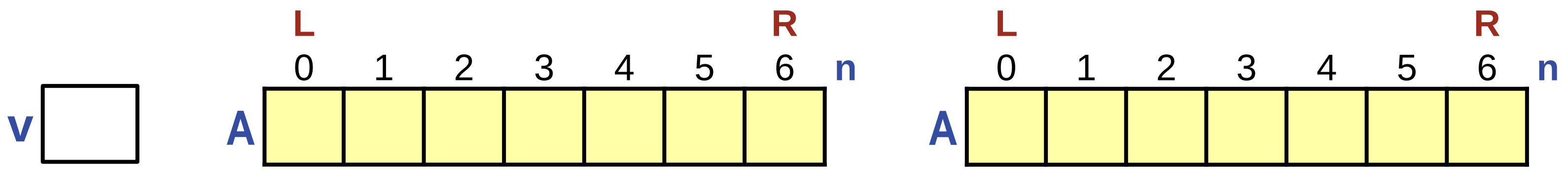

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/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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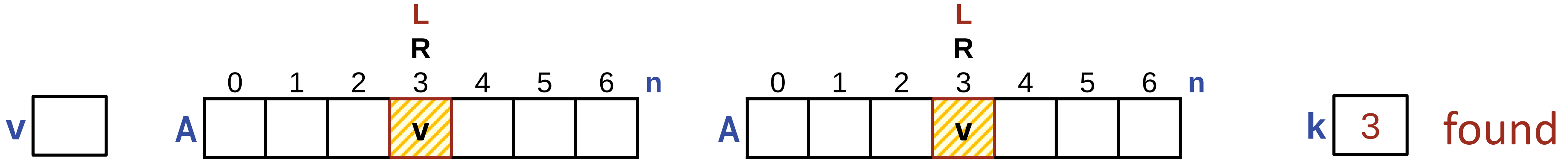

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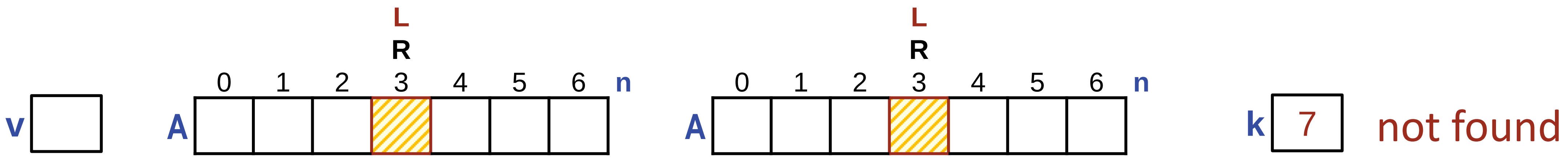
Binary Search



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Coding order
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v [ ] A (n=0) k [0] not found

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
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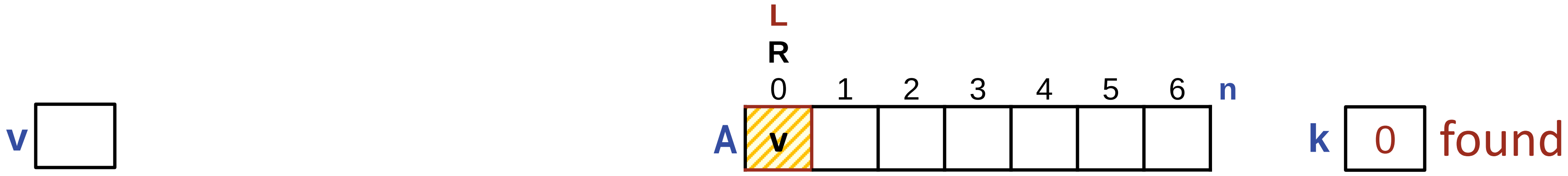

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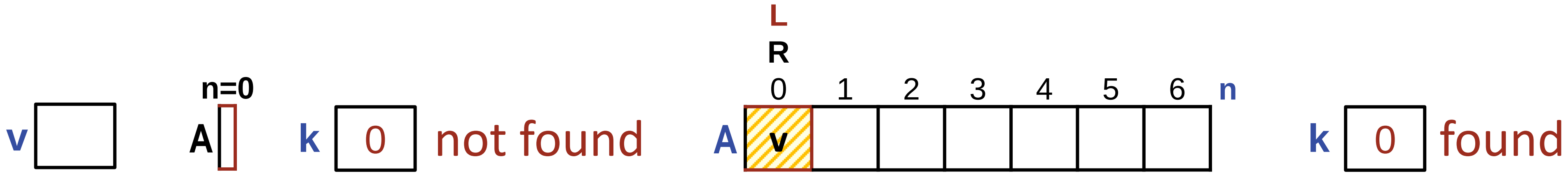

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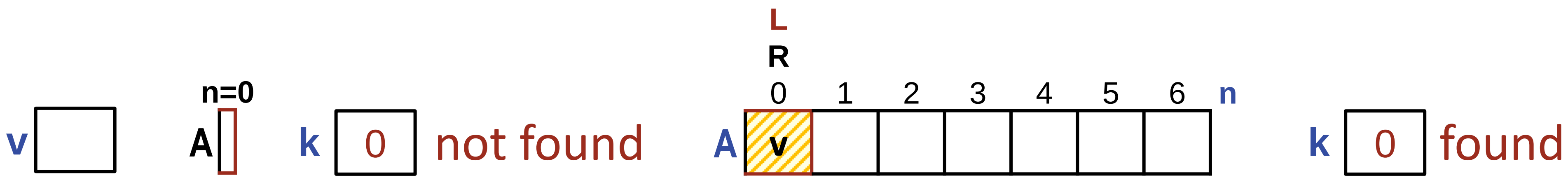
Is it a problem that k==0 represents both "not found" and "found in 0^{th} element"?

Binary Search



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            R = M;             // Select left “half”.
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      if ( A[L]==v ) k = L; else k = n;
      }
if ( k<n ) /* Found. */ else /* Not found. */
```

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(4) finalization
(5) boundary conditions

No. What matters is whether k<n, not whether k==0.

Binary Search is astoundingly fast. If n==512, just 9 iterations to termination!

Iteration #	VARIANT
0	512
1	256
2	128
3	64
4	32
5	16
6	8
7	4
8	2
9	1

Running time is logarithmic in n,

and independent of whether v is in A or not.