CS611 Sample Midterm

October 13, 1998

1. Let D be a pointed cpo (i.e., with a least element $\perp$) and let f be a continuous function from D to D. Take $d = \texttt{lub}\{f^i(\perp) | i >= 0\}$. That is, d is the least upper bound of the chain $\{\perp, f(\perp), f(f(\perp)), ...\}$.

   (a) Show that d is a fixed point of f.

   (b) Show that d is the least fixed point of f.

2. Briefly discuss the relative merits and shortcomings of large-step vs. small-step operational semantics.

3. One of the extensions to IMP we discussed was the addition of a division operation. i.e.

$$a := n \,|\, X \,|\, a_1 + a_2 \,|\, a_1 * a_2 \,|\, a_1/a_2$$

   Because of the possibility of division by zero, we modeled this in the small step operational semantics to allow arithmetic expressions, boolean expressions and commands to step to a new wrong value instead of a number, boolean value or a state. i.e.

$$< a, s > \rightarrow_1^* n + \texttt{wrong}_a$$

$$< b, s > \rightarrow_1^* \texttt{true} + \texttt{false} + \texttt{wrong}_b$$

$$< c, s > \rightarrow_1^* < \texttt{skip}, s' > + \texttt{wrong}_c$$

   We described this in the small step operational semantics with the addition of quite a number of new rules which detect and propagate these wrong values. A few of these rules are:

$$< n_1/0, \sigma > \rightarrow_1 \texttt{wrong}_a \qquad < n_1/n_2, \sigma > \rightarrow_1 \; n_3 \,(n_3 = n_1/n_2)$$

$$< \texttt{wrong}_a + a_2, \sigma > \rightarrow_1 \texttt{wrong}_a \qquad < n_1 + \texttt{wrong}_a, \sigma > \rightarrow_1 \texttt{wrong}_a$$

$$\frac{< a, \sigma > \rightarrow_1 \texttt{wrong}_a}{< x := a, \sigma > \rightarrow_1 \texttt{wrong}_c} \qquad < \texttt{wrong}_c; c_2, \sigma > \rightarrow_1 \texttt{wrong}_c$$

$$\frac{< b, \sigma > \rightarrow_1 \texttt{wrong}_b}{< \texttt{if}\, b\, \texttt{then}\, c_1\, \texttt{else}\, c_2, \sigma > \rightarrow_1 \texttt{wrong}_c}$$

   Sketch out how we could extend the axiomatic semantics to describe IMP with division.

4. Suppose we had defined the execution of arithmetic expressions as follows:

$$< n, \sigma > \not\to_1$$

$$< X, \sigma > \to_1 < \sigma(X), \sigma >$$

$$\frac{< a_1, \sigma > \to_1 < a_1', \sigma >}{< a_1 \,\texttt{op}\, a_2, \sigma > \to_1 < a_1' \,\texttt{op}\, a_2, \sigma >}$$

$$\frac{< a_2, \sigma > \to_1 < a_2', \sigma >}{< a_1 \,\texttt{op}\, a_2, \sigma > \to_1 < a_1 \,\texttt{op}\, a_2', \sigma >}$$

$$< n_1 \,\texttt{op}\, n_2, \sigma > \to_1 < n_3, \sigma >$$

Given the lemma (Confluence):

$\forall a, \sigma$ if $< a, \sigma > \to_1^* < a_1, \sigma_1 >$ and $< a, \sigma > \to_1^* < a_2, \sigma_2 >$ then there exists an $< a_3, \sigma_3 >$ such that $< a_1, \sigma_1 > \to_1^* < a_3, \sigma_3 >$ and $< a_2, \sigma_2 > \to_1^* < a_3, \sigma_3 >$

a) Argue that the deterministic and non-deterministic semantics are equivalent.

b) When we define an operational semantics why would we desire it to be deterministic?