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Developing Tensor Operations with an Underlying Group Structure

Tensor computations frequently involve factoring or decomposing a tensor into a sum of rank-1 tensors (CANDECOMP-PARAFAC, HOSVD, etc.). These decompositions are often considered as different higher-order extensions of the matrix SVD. The HOSVD can be described using the
n-mode product, which describes multiplication between a higher-order tensor and a matrix. Generalizing this multiplication leads to the contracted product, a type of multiplication between two higher-order tensors. However, the contracted product does not preserve the order of a tensor and it is therefore difficult to extend other concepts of linear algebra to tensors. We describe a type of tensor-tensor multiplication that has an underlying algebraic structure, in that the set of $n\times n\times n$ tensors forms an algebraic group. Another possible higher-order extension of the SVD emerges, but more importantly new algorithms for data compression result that utilize known efficient computational methods. Furthermore, this new framework provides a way to talk about higher-order extensions of QR factorization and eigenvalue decompositions. Ideally, these ideas attempt to bridge the gap between orthogonal decompositions and rank-revealing decompositions.