REIMAGINING CALCULUS, COMPUTATIONAL SCIENCE AND ENGINEERING, AND APPLIED MATHEMATICS

Study Points for Faculty and Administrators in Engineering

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The dissolution of the Department of Theoretical and Applied Mechanics (T&AM) poses new challenges for the delivery of Math 1910, 1920, 2930, and 2940, what we will refer to as the "x90 sequence." These courses have a combined enrollment of over 2900 students per year and effective staffing has been the joint responsibility of T&AM and the Department of Mathematics. With the majority of T&AM faculty initially transferring to the School of Mechanical and Aerospace Engineering (M&AE), it is natural for that unit to assume in the short term the College's responsibility for teaching calculus. However, x90 instruction is a College[1] matter; so, in the spirit of openness, it is essential for its faculty to understand and comment upon that component of the T&AM/ M&AE merger and to develop with colleagues in Mathematics a long-term vision. To be consistent with the mandate to "Reimagine Cornell", the dialog should go well beyond the formulation of an x90 business model. It should look hard at the role of mathematics in engineering, examine connections to physical science, biological science, and computer science, and determine how best to set the stage for 21st century curriculum in the fields. Because the intermingling of research and teaching is a Cornell tradition, the scope of the inquiry should also include an examination of graduate education in Computational Science and Engineering (CSE) and the role of the Center of Applied Mathematics (CAM).

We regret the realities that have prompted the budget crisis, but the silver lining is that Engineering and Mathematics have a unique opportunity to ``think big'' on behalf of the University. Below we present a set of interrelated study questions that we hope can serve as a framework for organizing a broad conversation. The "A" questions have more to do with academic issues while the "B" questions have more to do with administration and resource allocation. Our goal is to prompt the engagement of an informed faculty. To ensure easy access to relevant facts and figures, we include two appendices. Appendix A details the content of the x90 sequence while Appendix B summarizes the resources that have recently been required for its delivery. Because it involves faculty lines, TA lines, and two colleges, the ultimate decision on how mathematics instruction goes forward rests with the Deans of Engineering and Arts and Sciences and the Provost. These administrators need a clear picture of faculty sentiment and a balanced assessment of the options.

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[1] Throughout this document "College" refers to the College of Engineering.

A1. Is joint Engineering+Math oversight of x90 content worth the hassle of coordination?

For many years, the Engineering/Mathematics Liaison Committee (EMLC) has effectively dealt with x90 content and related organizational issues. It is composed of faculty from both Engineering and Mathematics. The CCGB is charged with overseeing College requirements and the Freshman-Sophomore/pre-major experience. Historically, the EMLC has set the tone for collaboration between Engineering and Mathematics in matters that concern x90 pedagogy and TA training.

It is important to stress the importance of having a proactive and broad-minded EMLC that publically shares its thinking. There is a wide range of opinion in the College regarding the role of applications and rigor in x90 instruction and these tensions need to be understood. In addition, each Engineering major has its wish list of topics that it would like to see included in the x90 sequence. The resulting pressures need to be intelligently managed and that requires critical eyes from both Engineering and Mathematics.

A2. How can we better integrate x90 instruction and computing?

Unless they receive AP credit in Computer Science, Engineering freshmen must take one of three Introductions to Computer Science. One of these explicitly attempts to integrate concepts from pre-calculus and Math 1910, e.g., trigonometry, limits, discretization, approximation, etc. Another makes use of elementary linear algebra and co-registration with Math 2940 is recommended. On the other side of the coin, Matlab computing is sometimes used in Math 2940.

Further integration of computing and mathematics at the freshman-sophomore level needs to be considered, subject to the constraint that syllabus modifications do not impose additional time pressures on the students and do not make it more difficult to attract faculty instructors. The issues are controversial; but the overall ambition is worthy, if we are to educate students to appreciate the interplay between mathematics and computing in the context of engineering.

A3. Should undergraduate instruction in Computational Science and Engineering (CSE) be enhanced across the College?

Computation has taken a place with experiment and theory as a mode of doing both engineering and science. Technical and scientific progress depends upon our ability to construct mathematical models and to affirm their correctness through experimentation and computer simulation. The heightened profile of computing in modern engineering must be reflected in our degree programs through courses that build a practical appreciation for reliable numerical software. CSE entails the development of new models, new ways of analyzing models and data, and new ways of interpreting the output from computations. Discrete mathematics together with systems that support visualization, parallel computing, and large-scale data management are part of the emerging infrastructure. It is our responsibility as teachers and researchers to torque the undergraduate curriculum accordingly. Engineering at Cornell is famous for its rock-solid foundation in science and applied mathematics. For our students to be leaders in the 21st century, the College must be equally renowned for its approach to CSE.

A4. In what ways can an organized approach to engineering mathematics at the undergraduate level improve the CSE research environment?

Working to coordinate undergraduate courses in computing and applied mathematics directly benefits graduate education and research across the sciences and engineering. It is an occasion to create common denominators between departments with respect to both course content and course staffing. Furthermore, a flourishing undergraduate CSE scene that successfully integrates calculus, applied mathematics, and computing would create a "broader impact" gold mine for NSF proposal-writers. Proposals to that agency require broader impact statements that connect the envisioned research to education and the national welfare.

A5. If additional resources are required for curriculum reform, then how might they be obtained?

This is an important question because it is unlikely that the university will redirect funds for this purpose in the current economic climate. On the other hand, an organization of proactive faculty could pool their talents and obtain NSF funding (cf. NSF 10-511 PRISM, for example) to renovate Engineering Mathematics and make the x90 sequence resonate better with CSE objectives. Integration of undergraduate courses in programming, scientific computing, and applied mathematics would be part of the effort. For such an endeavor to be successful it would be necessary to have administrators who share the vision and to have broad support across the faculty in Engineering and Mathematics.

In this regard, it is important to appreciate the entry-level course strain on Mathematics that is currently induced by the diversity of mathematics requirements across the University. Seed money to develop new courses sounds great, but what will be the long term "environmental impact" after the external funding runs out?

B1. What are the Financial Parameters?

When thinking about x90 administration it is important to keep in mind the variables F_M, F_E, and F where

- F_M = the number of Mathematics faculty-line equivalents that are allocated for x90 teaching. How will the Math Chair reason about this number and staff the blackboard positions?
- F_E = The number of Engineering faculty-line equivalents that are allocated for x90 teaching. How will the College reason about this number and staff the blackboard positions?
- $F = F_M + F_E$. Going to larger lectures affects this value as will general budget cuts. Who is the proposer and defender of this value?

The corresponding graduate TA allocation variables G_M, G_E, and G are also part of the picture. Current values can be deduced from Appendix B. Whether or not x90 faculty lines should be reapportioned between Engineering and Mathematics is a critical discussion point.

B2. What are some x90 oversight possibilities?

The Engineering and Mathematics Liaison Committee (EMLC) embodies the kind of intellectual oversight that is required to help ensure quality x90 instruction. The question is whether the EMLC should stay the same (Option I), be part of a larger structure whose responsibility includes the assignment of TAs and faculty to the courses (Option II), or serve as an empowered negotiating partner that interacts with the Department of Mathematics (Option III). The optimum oversight arrangement will most likely be a mix of these options.

Option I. Let the Director of M&AE and the Chair of Mathematics (or their delegates) administer the x90 sequence. With this framework, M&AE simply takes over the teaching role of T&AM, partnering with Mathematics on behalf of the College. The EMLC continues in its current form.

Advantages. This option is easy to implement since it represents the status quo and harmonizes with traditional Chair/Director authority. Because it can evolve with time, Option I can be thought of as a realistic “initial value”. There has been over one year of experience with this option and certain things are going well. For example, non-MA&E Engineering faculty have been teaching in the x90 sequence.

Disadvantages. A concern with this approach is that Mathematics and M&AE are large academic units each having a long list of terribly worthy undergraduate obligations, of which engineering calculus is just one. Could x90 resource allocation be adequately defended in a climate of tightening budgets? This would depend upon the inclinations of the relevant Deans, Chairs and Directors. These inclinations would vary with time forcing the creation of a complicated network of IOU’s. Could meaningful curriculum development take place in such an environment? How would M&AE and Mathematics react if (say) CS, OR, and ECE make a case for specialized version of 2940? What is to guarantee that faculty hiring in M&AE preserves over time a critical mass of x90-suitable applied mathematicians? With the retirements of former T&AM faculty, there will be pressure in M&AE to hire individuals who align with traditional M&AE interests, perhaps at the expense of College needs in applied mathematics. By choosing Option I, Engineering could inadvertently send a message that it has devalued the teaching/research importance of applied mathematics.

Option II. Create a field-like structure[2] for Computational and Applied Mathematics that would include faculty who are willing and able to teach in the x90 sequence and who would coordinate upper level instruction in applied math and computational science and engineering (CSE). A Director would have a measure of control over the resources necessary to deliver the x90 sequence and would be intellectually advised by the EMLC. Set protocols determined by the Provost would dictate financial interactions with Mathematics and Engineering.

Advantages. The advantage of this option is that it creates a strong, single voice for x90 instruction and provides a clubhouse for CSE innovation that transcends department boundaries. It would not necessarily reduce the <u>volume</u> of administration; teaching assignment and course scheduling negotiations would still be complicated. However, it could <u>simplify</u> the administration if the emerging

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[2] “Field-like structure” is a **<u>placeholder</u>**. It could be a center, a program, or some yet-to-be-imagined entity.

unit had a clear mandate from the Provost regarding the F and G parameters mentioned in B1. The Option II structure provides for longitudinal effort. Instead of x90 instructors complaining about high school preparation, upper-level instructors complaining about x90 instruction, and CSE researchers lamenting about the sorry state of undergraduate preparation, Option II gives Cornell the chance to formulate a unified, multilevel approach to these national issues.

Disadvantages. Teaching is generally assigned by departments. Coordination with the proposed unit would involve multiple department directors and chairs, each with a primary mission to look after their own curriculum. What incentive would they have to allow their faculty to participate? How sustainable would that be? How many faculty would be willing to join the proposed structure if it entailed substantial entry-level instruction? Most mathematicians in the Engineering College and in the Field of Applied Mathematics have no involvement whatsoever with x90 instruction. With respect to TA allocation and assignment, Option II requires some unrealistic decouplings. General TA-support is connected to graduate admissions and x90 staffing is connected to the staffing of upper level courses. How can these waters be navigated without significantly adding to the administrative burden of Chairs and Directors?

Option III. The Mathematics Department administers the x90 sequence in consultation with an EMLC, whose College members are appointed by the CCGB or the Dean.

Advantages. This Option has a certain simplicity, especially if College resources for teaching the x90 courses are transferred to the Mathematics Department. Such a resource shift does not preclude College faculty teaching these courses. There would be flexibility in teaching assignments across the applied mathematics curriculum because everything would be brought under "one roof," including the management of capable visiting instructors.

Disadvantages. Incentive(s) for faculty participation in teaching x90 courses will likely become secondary priorities in the College. Course offerings in applied mathematics might be at risk unless the EMLC was properly empowered and proactive. How receptive would Mathematics be to College requests for specialized versions of 2930 and 2940? There would be increased pressure to delegate x90 instruction to visitors.

B3. How can faculty be motivated to teach in the x90 sequence and in follow-up courses that support an undergraduate CSE initiative?

This is perhaps <u>the</u> central concern because it ultimately defines the quality of instruction. One possibility is to correlate faculty participation with TA-ships to the faculty member's home department. This would create a <u>budgetary</u> incentive and at the same time ensure that the caliber of teaching is high. Not to be dismissed is the <u>intellectual</u> incentive that would arise if the x90 courses and beyond are redesigned to capture the exciting interplay that exists between applied mathematics, computing, and Engineering research. That kind of curriculum development should be fostered by any new administrative arrangement.

B4. Are there cost-cutting ways to unify and improve the administration of CAM, CSE, and x90 sequence?

For nearly fifty years, the Center for Applied Mathematics (CAM) has attracted outstanding graduate students, often with backgrounds in pure mathematics, and provided them with the opportunity of working with Cornell faculty across a wide range of mathematical applications in science and engineering. More and more often, these applications have involved computational science.

The Center is home of the largest interdisciplinary Graduate Field at Cornell but has no faculty lines and no teaching responsibilities. Many of its graduate students have been supported as teaching assistants in courses taught by departments with close associations to the Center, such as Mathematics, Computer Science, and (in the past) T&AM. Under Option III, it would be particularly natural for the Department of Mathematics to assume the administration of the Center and its associated field. On the other hand, the Graduate Minor Field of Computational Science and Engineering is more likely to flourish if it is administered through a creative Option II structure.

Regardless, CAM students and CSE-minor students bring a lot to the table as TAs in the x90 sequence. Whatever administrative structure is adopted, it should promote easy access to this talent pool.

B5. What is at stake in the long term?

A reimagined administrative structure that successfully delivers new mathematical and CSE opportunities for Engineering students at both the undergraduate and graduate levels would further enhance Cornell's reputation as university that is particularly supportive of interdisciplinary study and research. The Faculty of Computing and Information Science is an example of a newly-formed unit that permits the University to track a rapidly changing research area that transcends colleges and departments. Engineering needs to be equally creative if it is to maintain high visibility in the coming years.

Appendix A: x90 Syllabi[1]

	Math 1910	Math 1920	Math 2930	Math 2940
1	Fundamental Theorem	Polar Coordinates	First order differential equations	Introduction/ Linear systems
2	Substitution in definite integrals	Conic sections	Initial value problems	Row reduction
3	Numerical integration	Vectors in a plane	Separable equations	Vectors, linear combinations
4	Areas between curves	Cartesian coordinates/vectors in space	Linear equations	Matrix equations
5	Volumes by slicing	Dot products	Exact equations	Solution sets of Ax = b
6	Volumes of revolution	Cross products	Mathematical models	Linear independence
7	Cylindrical shells	Lines and planes in space	Qualitative methods	Linear transformations
8	Curve length and surface area	Vector-valued functions	Numerical methods	Matrix of linear transformations
9	Inverse functions and derivatives	Modeling projectile motion	Linear differential equations	Applications
10	Natural logarithms	Arc length/unit tangent vector	Second order differential equations	Matrix operations, inverse
11	The exponential and other bases	Planetary motion	Constant coefficients/homogeneous eqna	Invertible matrices
12	Growth and decay	Functions of several variables	Complex roots	Partitioned matrices
13	L'Hospital's rule	Limits and continuity	Nonhomogeneous equations	Iterative solution of linear equations
14	Inverse trigonometric functions	Partial derivatives	Undetermined coefficients	Computer graphics
15	Hyperbolic functions	Differentiability/linearization	Phase plane analysis	Determinants
16	Basic integration formulae	The chain rule	Boundary value problems	Cramer's rule
17	Integration by parts	Directional derivatives/grad/tangent plane	Eigenvalue problems	Vector spaces
18	Partial fractions	Extreme values / saddlepoints	Linear systems of ODEs	Null and column spaces
19	Trigonometric substitution	Double integrals	Introductions to PDEs	Linear independence
20	Improper integrals	Applications: Center of mass	Fourier series	Coordinate systems
21	Limits of sequences of numbers	Integrals in polar coordinates	Sine and cosine series	Dimension
22	Theorems for limits	Triple integrals	Separation of variables	Rank
23	Infinite series	Spherical and cylindrical coordinates	Heat equation	Change of basis
24	Integral test	Line integrals	Wave equation	Applications
25	Comparison tests	Vector fields	Laplace's equation	Eigenvectors
26	Ratio tests	Flux and circulation		Diagonalization
27	Absolute convergence	Green's theorem		Linear transformations
28	Power series	Surface integrals		Complex numbers
29	Taylor and Maclaurin series	Stoke's theorem		Complex eigenvalues
30	Taylor series convergence	Divergence theorem		Discrete dynamical systems
31	Application of power series	Curl/potential functions		Applications to differential equations
32				Iterative estimates of eigenvalues
33				Orthogonal sets
34				Inner products
35				Orthogonal projection
36				Gram-Schmidt process
37				Least squares problems
38				Inner product spaces
39				Diagonalization of symmetric matrices

Notes: Each course is four credit hours and requires two prelims. The 1910/1920 format is three faculty-run lectures and two TA-run sections per week. The2930/2940 format is three faculty-run lectures and one TA-run section per week.

[1] We would like to thank the Department of Mathematics and the Engineering Advising Office for helping us assemble this table.

Appendix B. Recent Faculty/TA Allocations for Math 1910-1920-2930-2940[1]

Year	Course	#Students (Fall + Spr)	Fall								Spring							
			Math Resources				T&AM Resources				Math Resources				T&AM Resources			
			Lectures		Sections		Lectures		Sections		Lectures		Sections		Lectures		Sections	
2007-8	1910	398 + 51	2+4	4+6	6	12	2+4	2+4	2	4	1	1	1	2	0	0	0	0
	1920	432 + 396	7	12	7	15	3	3	0	0	1	2	8	16	0	0	0	0
	2930	420 + 438	1	1	2	6	1	2	4	12	1	2	1	3	0	0	4	12
	2940	401 + 415	1	2	3	9	0	0	2	6	0	0	5	15	2	3	1	3
	Total	1651 + 1300	9+4	25	15	42	6+4	11	8	22	3	5	15	36	2	3	5	15
2008-9	1910	439 + 48	4+2	6+4	8	16	1+4	2+4	0	0	1	2	1	2	0	0	0	0
	1920	447 + 417	7	14	6	13	3	3	2	4	1	2	7	14	0	0	1	2
	2930	433 + 417	1	1	2	6	2	2	4	12	1	2	2	6	0	0	4	12
	2940	350 + 440	2	2	3	9	0	0	1	6	0	0	3	10	2	3	2	6
	Total	1669 + 1322	14+2	27	19	44	6+4	11	7	22	3	6	13	32	2	3	7	20
2009-10	1910	401 + 37	2+4	3+5	5	11	1+3	2+3	1	2	1	2	1	2	0	0	0	0
	1920	472 + 385	7	11	5	10	3	3	2	4	1	2	7	14	0	0	0	0
	2930	415 + 449	0	0	3	9	2	2	3	9	1	2	2	6	0	0	4	12
	2940	368 + 378	2	2	3	9	0	0	2	6	0	0	3	9	2	2	2	6
	Total	1656 + 1249	11+4	21	16	39	6+3	10	8	21	3	6	13	31	2	2	6	18

Key: # Faculty in Red | #Grad TAs in Blue | #Classes they oversee in Black

Notes:

1. Typical 1910-1920 TA-ship = two sections that each meet twice a week.
2. Typical 2930-2940 TA-ship = three sections that each meet once a week.
3. 1910-1920 student load = three lectures and two sections per week.
4. 2930-2940 student load = three lectures and one section per week.
5. In 1910-1920, about 50% of lectures are covered by visiting faculty.
6. In 2930-2940, about 20%. of lectures are covered by visiting faculty.
7. Typically, about one T&AM TA-ship goes to a CAM student each semester.
8. Typically, about 1-in-4 Math TA-ships go to non-Math students.

We mention that faculty staffing levels for AY 2010-11 are at a somewhat lower level due to budget cuts. Small-section calculus will give way to small-lecture calculus with approximately 100 students per lecture.

[1] We would like to thank Michelle Klinger in the Department of Mathematics for helping us assemble this table.