Remembering Ralph Byers

Q. Then R. Then What?

The PhD Thesis of an Original Thinker

Charles F. Van Loan

Schooling Highlights

1977 Graduates from McGill. Math Major. (Nearly) Straight-A.

1977 Entered Cornell PhD program in Applied Mathematics.

1978 Took 2-semester sequence in Matrix Computations from Frank Luk.

1979 I become Ralph's PhD Advisor

1980 Begins work on Hamiltonian Problems

1982 Thesis: *Hamiltonian and Symplectic Algorithms for the Algebraic Riccati Equation*

Table Of Contents

Table Of Contents

The 2-by-2 Block Players

The Symplectics:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \Rightarrow \qquad S^{-1} = \begin{bmatrix} S_{22}^T & -S_{12}^T \\ -S_{21}^T & S_{11}^T \end{bmatrix}$$

The Hamiltonians:

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \qquad \Rightarrow \qquad H_{22} = -H_{11}^T,\ H_{12}^T = H_{12},\ H_{21}^T = H_{21}$$

The Transformers

General Orthogonal Symplectics:

$$Q = \begin{bmatrix} Q_1 & Q_2 \\ -Q_2 & Q_1 \end{bmatrix}$$

Householder Symplectics:

$$Q = \begin{bmatrix} I - 2uu^T & 0 \\ 0 & I - 2uu^T \end{bmatrix}$$

Givens Symplectics:

$$Q = \begin{bmatrix} \text{diag}(c_i) & \text{diag}(s_i) \\ -\text{diag}(s_i) & \text{diag}(c_i) \end{bmatrix}$$

Wanted

A structure-preserving QR iteration to compute the Hamiltonian Schur Decomposition:

$$\begin{bmatrix} Q_1 & Q_2 \\ -Q_2 & Q_1 \end{bmatrix}^T \begin{bmatrix} A^T & G \\ F & -A \end{bmatrix} \begin{bmatrix} Q_1 & Q_2 \\ -Q_2 & Q_1 \end{bmatrix} = \begin{bmatrix} T^T & G \\ 0 & -T \end{bmatrix}$$

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Ralph figured this out for the case rank(F) = 1, a.k.a., the single input optimal control case.

Symplectic Q. Then Symplectic R.

$$\begin{bmatrix} Q_1 & Q_2 \\ -Q_2 & Q_1 \end{bmatrix}^T \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T^T & G \\ 0 & -T \end{bmatrix}$$

Then What?

Hamiltonian QR Iteration:

$$Q^T \left[(H + \sigma I)(H - \sigma I)^{-1}\right] = R$$

$$H \leftarrow Q^T H Q$$

Ralph worked out:

- How to reduce H to condensed form.
- How to get Q without forming the symplectic $(H + \sigma I)(H - \sigma I)^{-1}$.
- How to choose shifts.
- How to deflate and reorder.

Classical Ralph

"I have something to show you. It may even be new."

–1981

Other Quotes

1977

“I have been able to apply my theoretical mathematics to write algorithms to solve real problems and analyze the results. I have also been able to use the computer to gain insight into theoretical problems.”

“My first love is ”pure” mathematics. However, there is a common ground between ”pure” mathematics and the practical world in applied mathematics. I hope to contribute to that field.”

1981

"I am now engaged in thesis research on the computation of matrix functions. One of my goals is a hopefully) stable algorithm for the calculation of a matrix logarithm. This problem arises in control theory (in the area of systems identification) and in the social sciences (in the study of Markov Models). *I have come to have some understanding* of the fundamental sensitivity of this problem."

1983

	$n = 5$	$n = 10$	$n = 20$
Laub's Method	41	223	1445
Hamiltonian- Schur Decomposition	48	178	965

(Time in Milleseconds)

"For larger problems, the advantage of the Hamiltonian-Schur decomposition is unmistakable."