If Copernicus and Kepler Had Computers

Charles Van Loan
Department of Computer Science
Cornell University
Hypothesis

“If a computer had been available to Copernicus, he would have been content to patch up the Ptolemaic system rather than propose a new model of the cosmos.”

A. Mowshowitz *The Conquest of Will*
The Planets Wander
A Theory that Doesn’t Cover the Facts

Mars moving uniformly on Earth-centered circle
A Theory that Tries to Cover the Facts

Play with rpm’s and radii
Sample Epicycles

\[ r_2 = r_1/4, \quad T_2 = T_1/6 \]

\[ r_2 = r_1/4, \quad T_2 = -T_1/6 \]

\[ r_2 = r_1/4, \quad T_2 = T_1/8 \]

\[ r_2 = r_1/4, \quad T_2 = -T_1/8 \]
The Ptolemaic System

80 circles for the known solar system
Three Circles

Earth

Mars
Four Circles

Earth

Mars
The Question Again

If Copernicus had a computer would he have been content to patch up the Ptolemaic model by adding more circles on circles until the model fit the data with sufficient accuracy?
A Musing--Not a Yes-No Question
People, Places, and Things

<table>
<thead>
<tr>
<th>People</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ptolemy</td>
<td>100AD</td>
</tr>
<tr>
<td>Copernicus</td>
<td>1473-1543</td>
</tr>
<tr>
<td>Tycho</td>
<td>1546-1601</td>
</tr>
<tr>
<td>Kepler</td>
<td>1571-1630</td>
</tr>
</tbody>
</table>

The Circle

The Ellipse

Sun
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Relevance

The current revolutions in biological science and network science rival what happened astronomy back in the 1500s.

There are Keplers amongst us.
Biology = Chemistry + Information

The cell is the computer.

DNA is the program

Describe what goes on in the cell in computer science terms.
Newton/Leibniz invented the calculus in order to explain planetary motion.

Computer scientists are inventing new mathematics to explain “network phenomena.”
Talk Outline

• The Kepler’s Laws and the Ellipse
• The Eccentric, the Equant, the Epicycle
• Building Models
• Explaining Mars
• Conclusion
Part I. The Ellipse

Kepler

Music of the Spheres
Kepler’s three laws of planetary motion each have a computational science “message”.

Here are #1 and #2:

1. Shape: Planetary orbits are elliptical.

2. Speed: The planet’s speed varies.
Eccentricity measures distance to "circle-hood"
Aphelion and Perihelion
## Perihelion, Aphelion, & Eccentricity

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$A$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>46.0032</td>
<td>69.8200</td>
<td>0.2056</td>
</tr>
<tr>
<td>Venus</td>
<td>107.4819</td>
<td>108.9441</td>
<td>0.0068</td>
</tr>
<tr>
<td>Earth</td>
<td>147.1053</td>
<td>152.1028</td>
<td>0.0167</td>
</tr>
<tr>
<td>Mars</td>
<td>206.6782</td>
<td>249.2205</td>
<td>0.0933</td>
</tr>
<tr>
<td>Jupiter</td>
<td>740.6071</td>
<td>816.5574</td>
<td>0.0488</td>
</tr>
<tr>
<td>Saturn</td>
<td>1353.6339</td>
<td>1513.3940</td>
<td>0.0557</td>
</tr>
</tbody>
</table>
The Inner Four Planets
Kepler’s Second Law
Orbital Speeds: Mercury - Venus - Earth
An Ellipse is the “Sum” of Two Circles
Ellipse “=“ Two Circles

\[ x(t) = a \cos(t) \]
\[ y(t) = b \sin(t) \]

\[ x(t) = \frac{a + b}{2} \cos(t) + \frac{a - b}{2} \cos(-t) \]
\[ y(t) = \frac{a + b}{2} \sin(t) + \frac{a - b}{2} \sin(-t) \]
Two Circles are Not Enough

A two-circle epicycle can trace out an ellipse, but it does not replicate the motion prescribed by Kepler’s Second Law.
The Motion is NOT Keplerian
Aside: Disproving a Model

Are these feasible equal-time snapshots?

Perihelion is in January so the Earth is closer to the Sun during southern hemisphere summer.
Hemisphere Views

Mar 22
Jun 22
Sep 22
Dec 22

Apr 22
Jul 22
Oct 22
Jan 22

May 22
Aug 22
Nov 22
Feb 22
Equal Angle Would Imply Hemispheric Energy Imbalance

\[
\frac{E_{\text{north}}}{E_{\text{Total}}} = 0.496
\]
Kepler’s Third Law

\[
\text{Period}^2 / \text{MeanDist}^3 = \text{Constant}
\]
# Mean Distance from the Sun

<table>
<thead>
<tr>
<th></th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle</td>
<td>56.674</td>
<td>108.211</td>
<td>149.583</td>
<td>226.955</td>
<td>777.655</td>
<td>1431.287</td>
</tr>
<tr>
<td>Simple</td>
<td>57.912</td>
<td>108.213</td>
<td>149.604</td>
<td>227.949</td>
<td>778.582</td>
<td>1433.514</td>
</tr>
<tr>
<td>Time</td>
<td>59.136</td>
<td>108.216</td>
<td>149.625</td>
<td>228.944</td>
<td>779.508</td>
<td>1435.740</td>
</tr>
</tbody>
</table>

What is the definition of “mean”? 
Part II. The Eccentric, the Equant, & the Epicycle

Circle-based models that (try to) explain non-uniform motion.
Dilemma Facing the Ancients

• Want:
  Earth at the Center
  Uniform Circular Motion

• Reality:
  Sun at the Center
  Nonuniform Elliptical Motion
The Eccentric Model

Ray RP rotates uniformly. Observer at O
Eccentric Angles

- Eccentric Motion
- Observer Angles
The Eccentric Lives On!
The Equant Model

Ray QP rotates uniformly. Observer at O
Equant Angles
Part III. Building Models

1. Gather Data
e.g., location of Mars each night for 10 years

2. Decide Upon a Model
e.g., two-circle epicycle

3. Determine the parameters of the Model
e.g., radii, rotation rates, initial angles
Early astronomy was dominated by circle-based models that were used to predict things like planet location.

Ptolemy, Copernicus, and Tycho were all “circle guys”.

Models
Tycho’s Model
The Copernican Model of Mercury
Example: The Season Length Problem

<table>
<thead>
<tr>
<th>Season</th>
<th>Start</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>March 21</td>
<td>8:48</td>
</tr>
<tr>
<td>Summer</td>
<td>June 21</td>
<td>3:14</td>
</tr>
<tr>
<td>Autumn</td>
<td>September 22</td>
<td>18:43</td>
</tr>
<tr>
<td>Winter</td>
<td>December 21</td>
<td>14:43</td>
</tr>
</tbody>
</table>
A Theory that Doesn’t Cover the Facts

Sun moves on Earth-centered circle: Equal Seasons
Season Length Via the Eccentric

Observer at eccentric point. Sn moves uniformly
Model Parameters

There are two model parameters to determine:

The tilt of the “crosshair”
The displacement of the center.
The Computer Can Help

- Automatic Gathering of Data
- Handling Model Complexity
- Determining Model Parameters
Click the minimizing alpha.
xc = -0.014, d = -0.014, alpha = 55.300      Error = 01d 06h 23m

Autumn
89d 20h 12m
91d 02h 35m

Summer
93d 15h 35m
92d 10h 18m

Winter
88d 23h 51m
90d 04h 37m

Spring
92d 18h 11m
91d 12h 20m
Click the minimizing \( xc \).
Click the minimizing alpha.
\(xc = -0.039, \ d = -0.039, \ \alpha = 77.097\)  \(\text{Error} = 00d\ 09h\ 18m\)

**Seasons and Dates:**
- **Autumn:** 89d 20h 12m, 89d 14h 44m
- **Summer:** 93d 15h 35m, 94d 00h 22m
- **Winter:** 88d 23h 51m, 88d 14h 33m
- **Spring:** 92d 18h 11m, 93d 00h 10m
Click the minimizing $x_c$. 

Model Error vs. $x_c$.
$xc = -0.034, \ d = -0.034, \ alpha = 77.396 \quad Error = 00d\ 01h\ 18m$

Autumn
89d 20h 12m
89d 19h 25m

Summer
93d 15h 35m
93d 16h 14m

Winter
88d 23h 51m
88d 22h 40m

Spring
92d 18h 11m
92d 19h 29m
Click the minimizing xc.
Click the minimizing alpha.
Click the minimizing alpha.
xc = -0.033, d = -0.033, alpha = 77.120  Error = 00d 00h 21m

Autumn
89d 20h 12m
89d 20h 26m

Winter
88d 23h 51m
88d 23h 40m

Spring
92d 18h 11m
92d 18h 29m

Summer
93d 15h 35m
93d 15h 14m
Click the minimizing $x_c$. 

![Graph showing a minimum at $x_c$ with Model Error on the y-axis and $x_c$ on the x-axis.](image)
Click the minimizing alpha.
xc = -0.033, d = -0.033, alpha = 77.054  
Error = 00d 00h 17m

Autumn
89d 20h 12m
89d 20h 29m

Summer
93d 15h 35m
93d 15h 18m

Winter
88d 23h 51m
88d 23h 37m

Spring
92d 18h 11m
92d 18h 26m
Part IV. Explaining Mars

Can we predict Mars’ location using an extended epicycle model?

Mars is “hard” because of its highly eccentric orbit and because it is nearby.
Eccentrics & Equants Can’t Simulate Retrograde Motion
Acceptable Error

1 minute of arc

The Moon is 30 minutes wide
The Epicycle Model
The Extended Epicycle Model
The Extended Epicycle Model
### 9-circle Fit of Mars from Earth

<table>
<thead>
<tr>
<th>Radii</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17.5164</td>
<td>inf</td>
</tr>
<tr>
<td>141.6411</td>
<td>686.9910</td>
</tr>
<tr>
<td>6.6122</td>
<td>343.4955</td>
</tr>
<tr>
<td>0.3037</td>
<td>228.9970</td>
</tr>
<tr>
<td>0.0557</td>
<td>171.7477</td>
</tr>
<tr>
<td>-93.3380</td>
<td>365.2500</td>
</tr>
<tr>
<td>-0.7290</td>
<td>182.6250</td>
</tr>
<tr>
<td>-0.0077</td>
<td>121.7500</td>
</tr>
<tr>
<td>-0.0021</td>
<td>91.325</td>
</tr>
</tbody>
</table>

Error $\leq 5.2$ minutes of arc
# 9-circle Fit of Venus from Earth

<table>
<thead>
<tr>
<th>Radii</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6478</td>
<td>inf</td>
</tr>
<tr>
<td>67.2405</td>
<td>224.7050</td>
</tr>
<tr>
<td>0.2272</td>
<td>112.3525</td>
</tr>
<tr>
<td>0.0011</td>
<td>74.9017</td>
</tr>
<tr>
<td>0.0001</td>
<td>56.1763</td>
</tr>
<tr>
<td>92.9481</td>
<td>-365.2500</td>
</tr>
<tr>
<td>0.7761</td>
<td>-182.6250</td>
</tr>
<tr>
<td>0.0081</td>
<td>-121.7500</td>
</tr>
<tr>
<td>0.0002</td>
<td>-91.325</td>
</tr>
</tbody>
</table>

Error $\leq 0.2$ minutes of arc
Part V. Conclusion

- The Computer can make us lazy
  
  “tweak the old model”

- The Computer can make us creative
  
  “think outside the box”

The special role of mathematics and computer science...
An Ellipse Has Two “Radii”
Formulas and Programs

Do the area and perimeter formulae for circles generalize to the ellipse?

Yes:
\[
Area = \pi \left( \frac{A + P}{2} \right) \sqrt{AP}
\]

No:
\[
Perimeter = 2\pi \sqrt{AP} \quad Perimeter = 2\pi \left( \frac{A + P}{2} \right)
\]
Estimating the Perimeter
Programs, Like Formulas, Are Used to Express What We Know

```matlab
function p = perimeter(A,P,n)
t = linspace(0,pi,n+1);
a = (A+P)/2;
b = sqrt(A*P);
x = a*cos(t);
y = b*sin(t);
dx = x(2:n+1)-x(1:n);
dy = y(2:n+1)-y(1:n);
p = sum(sqrt(dx.^2 + dy.^2));
```
The Three Vertices of Science

- Experimental Science
- Theoretical Science
- Computational Science