COM S 783 / OR&IE 634 Approximation Algorithms **Fall Semester, 2003**

Assignment 3: 10/20/03

Lecturer: Yuval Rabani *Due date: 11/10/03*

1. Give a algorithmic geometric proof that for instances with two commodities, the maximum concurrent flow equals the sparsest cut. (The maximum concurrent flow is the reciprocal of the minimum congestion concurrent flow, i.e., it is the maximum fraction of the demands that can be concurrently routed subject to the capacity constraints.) *hint:* Use the flow conservation formulation for maximum concurrent flow. Take its dual. Show that the dual can be interpreted as a metric on $\mathbb{R}^2$.

2. Give a spreading metric relaxation for BISECTION. Show a lower bound on its integrality gap.

3. Consider the shortest path metrics of an infinite family of bounded degree expander graphs.

   (a) Prove a lower bound of $\Omega(\log n)$ on the distortion of embedding these metrics into ℓ_1, where n is the number of points in the metric.

   (b) Prove a similar lower bound for embedding these metrics into a distribution over dominating tree metrics.

4. An instance of MIN-CUT LINEAR ARRANGEMENT has as input an undirected graph $G = (V, W)$ with non-negative edge weights $w : E \to \mathbb{Q}^+$. The objective is to find a linear arrangement of minimum *cutwidth*. The cutwidth of a linear arrangement is the maximum number of edges that cross any given point on the line. (Clearly, checking a set of points $\xi_i \in (i, i+1)$, for all $i = 1, 2, \ldots, |V| - 1$, is sufficient.) Design and analyze an approximation algorithm for this problem.

   hint: Aim at an $O(\log^2 |V|)$ approximation guarantee.