COM S 783 / OR&IE 634 Approximation Algorithms	**Fall Semester, 2003**

Assignment 2: 9/29/03

Lecturer: Yuval Rabani *Due date: 10/13/03*

1. Vizing's theorem states that $\chi'(G) \leq \max_{x \in V(G)} \Delta(x) + m(x)$, where $\Delta(x)$ is the degree of x and $m(x)$ is the maximum multiplicity of an edge adjacent to x. Shannon's theorem states that $\chi'(G) \leq \left\lfloor \frac{3}{2}\Delta \right\rfloor$, where $\Delta = \max_{x \in V(G)} \Delta(x)$. Deduce Shannon's theorem from Vizing's theorem.

2. A MAXIMUM COVERAGE instance takes as input m explicit subsets of an n element ground set, and an integer $k \leq m$. The output is k of the input subsets, with the objective of maximizing the number of covered elements. The greedy algorithm iterates k times picking a subset that covers the most new elements. Analyze the performance guarantee of the greedy algorithm.

3. Propose LP relaxations for SET COVER and for MAXIMUM COVERAGE. Design randomized rounding-based approximation algorithms for both problems and analyze their approximation guarantees.

4. A HITTING SET instance has the same input as SET COVER. The goal is to find a minimum cardinality subset of the ground set (not necessarily one of the input subsets) that has non-empty intersection with each of the input subsets. A DOMINATING SET instance takes as input an undirected graph. The desired output is a minimum cardinality set of nodes that *dominates* all the nodes; i.e., every node either is in the dominating set or is a neighbor of a node in the dominating set. In the STEINER ARBORESCENCE problem, the input is a directed arc-weighted graph, a root node r and a set of terminal nodes T. The goal is to find a minimum weight arborescence (directed tree) rooted at r and spanning T.

   (a) For at least two of the problems, prove that they are at least as hard to approximate as SET COVER.

   (b) For at least two of the problems, propose approximation algorithms.

5. Prove that MULTICUT is at least as hard to approximate as VERTEX COVER.

6. Consider a variation of MULTIWAY CUT where for every non-terminal x there is a list of forbidden terminals T_x, and x cannot be in the same component as any terminal in T_x.

   (a) Propose a linear programming relaxation for this problem.

   (b) Analyze the integrality gap of your proposed relaxation for $k = 3$.