Motivation

• How might you solve the following?
  • Relight scene with novel illumination
  • Render image from novel viewpoint
  • Extract scene’s illumination
Methods we’ve seen

• Recover geometry
• Infer materials
• Render the scene

• Question: What if we have direct access to the scene at one point?
  • Fewer heuristic methods
  • Actual, physical measurements
  • What would we measure?
Light Transport

- Relation between incident and exitant light of static scene
- Linear
  - Simple linear photon model
  - No interference or diffraction
- Simply, light sums
- All bounces effectively summed
Transport Tensor

\[ \mathbf{E} = \mathbf{T} \mathbf{I} \]

- Exitant light
- Incident light

4D tensor
8D tensor
4D tensor
Why 4D?

• Why not 5D or 6D?
• x, y, z, normal - 5 parameters
• Same anywhere along normal (up to a scale factor) - eliminates 1 parameter
• Consider surface of convex hull only - surface coordinates + normal - 4 parameters
• Consider both incident and exitant to get 4+4 = 8D total
Transport Matrix

Exitant light

\[ E = TI \]

Incident light

For convenience, flatten out
Rendering Equation

\[
L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t)L_i(x, \omega', \lambda, t)(-\omega' \cdot n)d\omega'
\]
\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t)(-\omega' \cdot n)d\omega' \]
Rendering Equation

\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t)(-\omega' \cdot \mathbf{n})d\omega' \]
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- **Light out**
- **Emitted light**
- **BRDF**
- **Light in**
\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t)(-\omega' \cdot n) d\omega' \]

**Light out**

**Emitted light**

**BRDF**

**Light in**

**Attenuation**
Rendering Equation

\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t)(-\omega' \cdot n) d\omega' \]

Multiple bounces means \( L_o \) becomes \( L_i \)
Now we have a recurrence relation
Informal Explanation

\[ L_r(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t)L_r(x, \omega', \lambda, t)(-\omega' \cdot n) d\omega' \]

Turn the integral into infinite sum

\[ L_r(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \sum_{\omega' \in \Omega} f_r(x, \omega', \omega, \lambda, t)L_r(x, \omega', \lambda, t)(-\omega' \cdot n) \]
Informal Explanation

\[ L_r(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \sum_{\omega' \in \Omega} f_r(x, \omega', \omega, \lambda, t)L_r(x, \omega', \lambda, t)(-\omega' \cdot n) \]

Think of the function as vectors of infinite length

\[ L_{x,\omega} = E_{x,\omega} + \sum_{\omega' \in \Omega} (-\omega' \cdot n)F_{x,\omega',\omega}L_{x,\omega'} \]
Informal Explanation

\[ L_{x,\omega} = E_{x,\omega} + \sum_{\omega' \in \Omega} (\omega' \cdot n) F_{x,\omega',\omega} L_{x,\omega'} \]

\[ L = E + KL \]

\[ L - KL = E \]

\[ (I - K)L = E \]

\[ L = (I - K)^{-1} E \]

\[ L = TE \quad \text{Our familiar form} \]
Significance

\[ T = (I - K)^{-1} \]

\[ (I - K)^{-1} = I + K + K^2 \ldots \]

BRDF + attenuation rolled together

1st

Emission Direct bounce

\[ L = E + KE + K^2 E \ldots \]
Concrete Example

Exitant light (camera)

C = TL

Incident light (projector)

A 4D slice (no variation in direction)
Scene Relighting

We’ve already acquired $T$

Plug in a novel illumination

$C' = TL'$

And we have the relit scene
Helmholtz Reciprocity

Swap camera and illumination

\[ C'' = T^T L'' \]

Just transpose light transport

Note: \( T^T \neq T^{-1} \) due to light absorption and scattering.
When does it hold?

- When the BRDF is symmetric (swap incident and reflected directions)
- Enforced
Dual Photography
Pradeep Sen et al.

- Capture T
- Synthesize projector’s view with Helmholtz reciprocity
Acquiring T

- **Example:** 2D projector - photosensor

- \( T \) is \( 1 \)-by-\( mn \) where projector resolution is \( m \)-by-\( n \)

- **Brute force approach**
Acquiring $T$

- Example: 2D projector - 1D photosensor
- $T$ is 1-by-$mn$ where projector resolution is $m$-by-$n$
- Brute force approach

Until you have $T$
Relighting

- Specify novel I

\[
\begin{align*}
1.3 & = 0.7 & 0.5 & 0.3 \\
C & & T & \\
\end{align*}
\]
Dual Photo

- Apply Helmholtz reciprocity
- Projector -> camera
- Photosensor -> point light source

\[
\begin{bmatrix}
0.7 \\
0.5 \\
0.3
\end{bmatrix}
\begin{bmatrix}
0.7 \\
0.5 \\
0.3
\end{bmatrix}^T = 1
\]
Efficiency

- For m-by-n projector and p-by-q camera brute force approach
  - mn images
  - 15 megapixel camera, VGA projector, 24 bit color depth = ~12.5 TB per scene (no compression)

- Assuming 1 sec per image (exposure, storage, processing) ~85 hours to acquire

  *Multiplexing approach is necessary*
Adaptive Multiplexed Illumination

- Subdivide illumination space
- Check for conflicts in camera space
- If no collision, measure with both illuminations and sort out the separation later
- Degrades to brute force in complex scenes
Example
Example
Personal Experiment

- Rendered 128x128 images with 128x128 projector
- Used brute force approach
- Performed relighting and dual photography
- Artifacts present in dual image (Renderer light sampling?)
Personal Experiment

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- Used brute force approach
- Performed relighting and dual photography
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Friday, September 30, 11
Personal Experiment

Projector must have been upside down...oops
Personal Experiment

Projector must have been upside down...oops
Sparsity Observation
T is data sparse

• For m-by-n projector and p-by-q camera, the 4D slice still takes $O(mnpq)$ bytes to store (assuming no compression)

• Let’s find ways to exploit data sparsity
  • Sparse entries
  • Low rank approximations
  • Compressible basis transformations
Next Question

• Is the light transport data-sparse even more so in another basis?

• Idea: Let’s try a basis used for image compression - wavelets

• Why wavelets? Why not just a Fourier basis?

• Localized in both frequency and space
Wavelet Environment Matting

Pieter Peers et al.

The image \( \times \) \( C_1 \) \( = \) \( C_1 \times \) \( \text{composited image} \)

The image \( \times \) \( C_2 \) \( = \) \( C_2 \times \)

The image \( \times \) \( C_3 \) \( = \) \( C_3 \times \)

The image \( \times \) \( C_n \) \( = \) \( C_n \times \)

...
How it works

- Brute force: $C = TI$ (columns of identity form single pixel patterns)
- Turn it into $C = T(BB^T)I$
- $C = (TB)(B^TI)$
- $C = (TB)B^T$
What does that mean?

\[ C = (TB)B^T \]

Use vectors of basis as illumination patterns

basis vectors

\[ \times \]

\( x \)

\( C_1 \)

\( C_2 \)

\( C_3 \)

\( \ldots \)

\( C_n \)

composited image

Friday, September 30, 11
What does that mean?

And measure $T$ projected onto $B$ instead

$C = (TB)B^T$

basis vectors

$X \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots \\ 0 & 0 \end{pmatrix} = C_1$

$X \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ \vdots \\ 0 & 0 \end{pmatrix} = C_2$

$X \times \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = C_3$

$\vdots$

$X \times \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 1 \end{pmatrix} = C_n$

composited image
What does that mean? Let \( C' = (TB)(l'^TB)^T \). So if we want a novel illumination, project into the same basis.
Why do we care?

• At first glance, same number of captures as brute force
• More than 1 pixel illuminated - better SNR
• Under-sampling
  • Wavelets exploit spatial relationships
  • Might be better than discarding illumination pixels
Choosing wavelet basis vectors

Uses a feedback algorithm to choose next best vector
Results

Reference image

1000 Haar patterns

1000 Daubechies (9,7) patterns
Video Results
Video Results
Main take-away

• Haar wavelet basis is good for measuring $T$

• Or at least they contrived their test scenes well enough to be convincing
Compressive Light Transport Sensing
Pieter Peers et al.

- Bypass this whole wavelet basis vector selection and use compressive sensing theory

- We had $C = T(BB^T)I$ where $B$ is the Haar wavelet basis

- In reality we didn’t use all of $I$

- $C = (TB)(B^TA)$ where $A$ is subset of $I$’s columns
How it works

• Measure rows of $T$ one at a time

• $c_i = (t_i B)(B^T A)$

• $c_i^T = (A^T B) (B^T t_i^T)$

• Last paper showed empirically is sparse

• CS theory applies
CS Theory - High level

- Ignoring important properties like how to select A...
- $c_i^T = (A^T B)(B^T t_i^T) \Rightarrow y = \phi^T x$
- But x is sparse
- Want to solve $\arg\min_x ||x||_0 \text{ s.t. } y = \phi^T x$
- NP-complete
- Settle for $\arg\min_x ||x||_1 \text{ s.t. } y = \phi^T x$
- Linear programs - no strongly polynomial algorithm known
- Basis pursuit, orthogonal matching pursuit, ROMP, CoSaMP, etc.
Results
Compressive Sensing Experiment

150 samples
20 wavelet coefficients
One last idea

- All these methods acquire T
- Can we compute without acquiring T directly?
Optical Computing for Fast Light Transport Analysis

O’Toole et al.

• Krylov subspace methods
  • Arnoldi
  • GMRES

• Wherever you see $T_l$ or $T^{T_l}$, replace with black box physical process
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Numerical objective</th>
<th>Step 1</th>
<th>Step 4</th>
<th>Step 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power iteration (Section 2.1)</td>
<td>estimate principal eigenvector of T</td>
<td>(l_1 = \text{positive vector})</td>
<td>(l_{k+1} = p_k / |p_k|_2)</td>
<td>return (l_{k+1})</td>
</tr>
<tr>
<td>Arnoldi (Section 3)</td>
<td>compute rank-(K) approximation of (T)</td>
<td>(l_1 = \text{non-zero vector})</td>
<td>(l_{k+1} = \text{ortho}(l_1, \ldots, l_k, p_k)) (l_{k+1} = l_{k+1} / |l_{k+1}|_2)</td>
<td>return ([p_1 \cdots p_K][l_1 \cdots l_K]^T)</td>
</tr>
<tr>
<td>Generalized minimal residual (Section 4)</td>
<td>find vector (l) such that (p = T l)</td>
<td>(l_1 = \text{target photo} p)</td>
<td>(l_{k+1} = \text{ortho}(l_1, \ldots, l_k, p_k)) (l_{k+1} = l_{k+1} / |l_{k+1}|_2)</td>
<td>return ([l_1 \cdots l_K][p_1 \cdots p_K]^{+} p)</td>
</tr>
</tbody>
</table>
Optical Arnoldi Results
Optical Arnoldi Results
Optical GMRES Results
Optical GMRES Results
Power Iteration Example
Questions?
Exploit Symmetry

- So far
  - Considered a slice of 8D reflectance field
  - Point camera, 2D projector - 2D slice
  - 2D camera, 2D projector - 4D slice

- Full 8D reflectance field is symmetric
  - From Helmholtz reciprocity
Symmetric Photography

Gaurav Garg et al.

- Key idea: Represent $T$ as hierarchical tensor
- i.e. Don’t flatten into giant matrix...preserve locality
- Leaf nodes are rank-1
- Apply previous approaches to higher rank components
Illustration - 2D version

\[
\begin{bmatrix}
U_1 & M \\
M^T & U_2
\end{bmatrix} = \begin{bmatrix}
U_1 & \cdot \\
\cdot & U_2
\end{bmatrix} + \begin{bmatrix}
\cdot & \cdot \\
\cdot & M^T
\end{bmatrix}
\]

Idea - Measure flood light pattern, then subtract off-diag blocks
Illustration - 2D version

Illuminate all in parallel
Then decide if the off-diag block is rank-1
Illustration - 2D version

\[ r = M^T p_r \]
Next Steps

• Choose $p_c$ and $p_r$ to be the 1-vector (to sum cols and rows of $M$)

• Tensor product of $r$ and $c$ form $M$

• Check RMS error of low rank approximation

• If below threshold, label as leaf

• Else recurse
Illustration - 2D version

Now we need the diagonal blocks
Illustration - 2D version

\[
\begin{bmatrix}
U_1 & M \\
M^T & U_2
\end{bmatrix}
= 
\begin{bmatrix}
U_1 & M \\
M^T & U_2
\end{bmatrix}
+ 
\begin{bmatrix}
M \\
M^T
\end{bmatrix}
\]

We now can subtract off off-diag blocks ... and divide and conquer on \(U_1\) and \(U_2\)
Experimental Apparatus
Relighting Example
Relighting Example
Comparison to Sen

- Garg captures full 8D reflectance field
- Sen captured up to 6D slices
- Sen degrades to single pixel illumination
- Garg does as well, but exploits data-sparsity to prevent it (better SNR)
- Garg quality degrades with projector-camera misalignment - bit blurry
- Qualitatively, all looks the same to me