Call-by-name, call-by-value, and the λ-calculus

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Call-by-name vs call-by-value

Define square = λxy.x*x

Evaluate square(2+2, 2+3)

Call-by-name (CBN)	Call-by-value (CBV)
square(2+2, 2+3)	square(2+2, 2+3)
(2+2)*(2+2)	square(4, 2+3)
4*(2+2)	square(4, 5)
4*4	4*4
16	16

Termination under CBN but not CBV

$\Omega = (\lambda x.xx)\ (\lambda x.xx)$

CBN	CBV
$(\lambda xy.y)\ \Omega\ z$	$(\lambda xy.y)\ \Omega\ z$
$\lambda y.y\ z$	$(\lambda xy.y)\ \Omega\ z$
z	...

Objective

- Transformation between CBV and CBN semantics
- A CBV evaluation of a program *P* should terminate if and only if the CBN evaluation of the translated *P* also terminates

Theorem 2. (Simulation). $\Psi(\mathrm{Eval}_V(M)) = \mathrm{Eval}_N(\overline{M}(\lambda x x))$, for any program M.

Some Context

Published in 1975

Previous Papers:

Recursive Functions (1960)
Axiomatic Basis (1969)

Abstraction (1983)
Expressive Power of PLs (1990)

Higher-level languages:

C	1972
Scheme	1973
ML	1975

Practicalities

<S, E, C, D> machine

- Machine for evaluating lambda expressions

Constapply(a, b)

- Mechanism for syntactic sugar

The λ_v Calculus

$$\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{e \to e'}{v\ e \to v\ e'}$$

$$(\lambda x.e)\ v \to e\ [v/x]$$

I1. $(\lambda x M) = (\lambda y\, [y/x]\, M)\ (y \notin FV(M))$. **(α-rule)**

2. $(\lambda x M)\, N = [N/x]\, M$ **(if** N **is a value). (β-rule)**

3. $(ab) =$ **Constapply** (a, b) **(if this is defined). (δ-rule)**

Equality of terms

II1. $M = M$

2. $$\frac{M = N \quad N = L}{M = L}$$

3. $$\frac{M = N}{N = M}$$

III1. $$\frac{M = N}{(MZ) = (NZ)}, \qquad \frac{M = N}{(ZM) = (ZN)}$$

2. $$\frac{M = N}{(\lambda x M) = (\lambda x N)}$$

M=N iff M is equivalent to N

Reduction of terms

M≥N iff M reduces to something equal to N

Theorem 2. (Church–Rosser theorem). *If* $\lambda_V \vdash M_1 \geqslant M_i$ $(i = 2, 3)$ *then for some* M_4, $\lambda_V \vdash M_i \geqslant M_4$ $(i = 2, 3)$.

Theorem 4. $Eval_V(M) = N$ *iff* $M \overset{*}{\underset{V}{\rightarrow}} N$, *(for closed M and a value N)*.

The λ_n Calculus

$$\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2}$$

$$(\lambda x.e_1)\ e_2 \to e_1\ [e_2/x]$$

I1. $(\lambda x M) = (\lambda y\,[y/x]\,M)\,(y \notin FV(M))$ (α-reduction).

2. $(\lambda x M)\,N = [N/x]\,M$ (β-reduction).

No requirement that N is a value

Translating from CBN to CBV

(λxy.y) Ω z

$$\underline{x} = x$$

$$\underline{\lambda x.M} = \lambda\alpha.\alpha(\lambda x.\underline{M})$$

$$\underline{M\ N} = \lambda\alpha.\underline{M}\ (\lambda\beta.\beta\ \underline{N}\ \alpha)$$

Translating from CBN to CBV

$$(\lambda xy.y\ \Omega)\ z$$

$$\underline{x} = x$$

$$\underline{(\lambda x M)} = \lambda \varkappa \varkappa (\lambda x \underline{M})$$

$$\underline{(MN)} = \lambda \varkappa \underline{M} (\lambda \alpha \alpha \underline{N} \varkappa).$$