

```

| ⊢ ∀D:ℙ. ∀A:D → ℙ. (∃x:D. (A x) ⇔ ∀X:ℙ. ((∀x:D. ((A x) ⇒ X)) ⇒ X))
|
| BY RepeatFor 4 ((D 0 THENA Auto))
| \
| 1. D: ℙ
| 2. A: D → ℙ
| 3. ∃x:D. (A x)
| ⊢ ∀X:ℙ. ((∀x:D. ((A x) ⇒ X)) ⇒ X)
| |
| 1 BY (D 0 THENA Auto)
| |
| 4. X: ℙ
| ⊢ (∀x:D. ((A x) ⇒ X)) ⇒ X
| |
| 1 BY (D 0 THENA Auto)
| |
| 5. ∀x:D. ((A x) ⇒ X)
| ⊢ X
| |
| 1 BY D 3
| |
| 3. x: D
| 4. A x
| 5. X: ℙ
| 6. ∀x:D. ((A x) ⇒ X)
| ⊢ X
| |
| 1 BY (InstHyp [x] 6. THENA Auto)
| |
| 7. X
| ⊢ X
| |
| 1 BY Hypothesis
| \
| 1. D: ℙ
| 2. A: D → ℙ
| 3. ∀X:ℙ. ((∀x:D. ((A x) ⇒ X)) ⇒ X)
| ⊢ ∃x:D. (A x)
|
| BY BHyp 3
| \
| ⊢ ∃x:D. (A x) ∈ ℙ
| |
| 1 BY Auto
| \
| ⊢ ∀x:D. ((A x) ⇒ (∃x:D. (A x)))
|
| BY RepeatFor 2 ((D 0 THENA Auto))
|
| 4. x: D
| 5. A x
| ⊢ ∃x:D. (A x)
|
| BY (InstConcl [x]. THENA Auto)
|
| ⊢ A x
|
| BY Hypothesis

```

$\lambda D, A. \langle \lambda ex, X, all. \text{let } x, ax = ex \text{ in all } x \text{ ax}, \lambda ex. (ex (x:D \times (A x)) (\lambda x, all. \langle x, all \rangle)) \rangle$