Logic of Programs | **Lecture 7**
CS 6860 Fall 2015 | **Tues. Sept 15, 2015**

Lecture 7

Today's lecture will explore the *computational interpretation* of the two constructive logics, mPC and iPC and contrast that with evidence for the axiom of classic PC, $P \vee \sim P$.

Chapter 2 of the recommended textbook, *Type Theory and Functional Programming* by Simon Thompson, provides an account of these ideas based on a Natural Deduction style for proofs. We have discussed this style briefly and compared it to our Refinement style sequent calculus.

For Lecture 8 on Thursday, you should read Per Martin-Löf's "On the meanings of the logical constants." They provide a philosophical approach to the topics of this lecture.

This lecture will discuss a research issue about iPC and a new conjecture I have about the computational interpretation of iPC.

The lecture notes for Lecture 8 will be the Martin-Löf article.

Computing in iPC, the rules for any(t)

The proof rule for *ex falso quodlibit* (False elimination) has an extract, any(t).

$$H,\ x\!:\!\text{False},\ H' \vdash G \text{ by any}(x).$$

We have in the past not attempted to compute with any(t). There were no computation rules. Here we propose a rule and examine its behavior. The detailed notes will not be posted since we want to study this phenomenon first. Here are two interesting examples:

$\vdash (A \Rightarrow B) \Rightarrow (\sim C \Rightarrow (C \Rightarrow B))$	$\lambda(ab.\lambda(nc.\lambda(x.\_\_\_\_\_)))$
$ab\!:\!A \Rightarrow B,\ nc\!:\!C \Rightarrow \text{False},\ x\!:\!C \ \vdash B$	by $\text{ap}(nc;\ \_\_\_;\ \_\_\_v.\_\_\_)$
$\vdash C$	by x
$v\!:\!\text{False} \ \vdash B$	by $\text{any}(v)$
	Note $v = \text{ap}(nc; x)$

$\lambda(ab.\lambda(nc.\lambda(x.\text{ap}(nc; x; v.\text{any}(v)))))$

Note $\text{ap}(nc; x; v.\text{any}(v))$ reduces to $\text{any}(\text{ap}(nc; x))$.

So the extract is $\lambda(ab.\lambda(nc.\lambda(x.\text{any}(\text{ap}(nc; x)))))$

Here is another proof of $\vdash (A \Rightarrow B) \Rightarrow (\sim C \Rightarrow (C \Rightarrow B))$

	$\vdash (A \Rightarrow B) \Rightarrow (\sim C \Rightarrow (C \Rightarrow B))$	$\lambda(ab.\lambda(nc.\lambda(x.\_\_\_\_\_\_)))$
$ab\!:\!A \Rightarrow B,\ nc\!:\!C \Rightarrow \text{False},\ x\!:\!C$	$\vdash B$	by $\text{ap}(ab;\_\_\_;\_\_\_v.\_\_\_)$
	$\vdash A$	by $\text{ap}(nc;\_\_\_;w.\_\_\_)$
	$\vdash C$	by x
$w\!:\!\text{False}$	$\vdash A$	by $\text{any}(w)$
$v\!:\!B$	$\vdash B$	by v
		Note v is $\text{ap}(ab;\_\_\_)$

The evidence term is $\lambda(ab.\lambda(nc.\lambda(x.\text{ap}(ab;\text{any}(\text{ap}(nc;x))))))$

How to compute with $\text{any}(\text{ap}(nc;x))$?

That gives $\lambda(ab.\lambda(nc.\lambda(x.\text{any}(\text{ap}(nc;x)))))$.

This is the same as $\lambda(ab.\lambda(nc.\lambda(x.\text{any}(\text{ap}(nc;x)))))$!