Logic of Programs	**Lecture 17**
CS 6860 Fall 2015	**Thurs. Oct 22, 2015**

Lecture 17

The article by Kreisel (JSL vol. 27: 139-158, 1962) posted with Lecture 16 also deals with completeness issues for iFOL. Additionally he treats several conceptual issues about the logic of intuitionistic and constructive mathematics. One of these issues is the treatment of abstract notions such as computability. He even draws attention to the concept of the Euclidean plane, noting that it should not be identified with coordinate names for points (p. 122). Among the abstract notions discussed in this paper is the status of *Church's Thesis.* He writes it as

$$\forall f.\exists e.\forall n.\exists p.[T(e, n, p)\, \&\, f\, n = U(p)]$$

Here f is a computable function from natural numbers to natural numbers, e is a natural number that codes the algorithm defining f and p is the code of a computation of f on input n with function U picking out the value of f on n from the code p of the computation. The T predicate is Kleene's famous "T Predicate", a primitive recursive function that specifies that p *codes the computation* of f on input n.

Kreisel asks what are the *logically significant properties* of computable functions and their computations (p. 123). The key conviction motivating his work is the belief that the mental experience of (constructive) mathematical activity is amenable to analysis.

We will also look at Kreisel's 1962 JSL article, *On Weak Completeness of Intuitionistic Predicate Logic*, 139-158. He defines completeness as the following assertion, written here in the notation of Constructive Type Theory.

$$\begin{array}{l} \forall D\!:\!\text{Type.}\ \forall P_1\!:\!D^{n(1)} \rightarrow \text{Prop}, ..., \forall P_k\!:\!D^{n(k)} \rightarrow \text{Prop.} \\ (F(P_1, ...P_k) \Rightarrow \exists p\!:\!\mathbb{N}.\text{Prov}(p, \ulcorner F \urcorner)) \end{array} \tag{1}$$

where $n(i)$ is the arity of propositional function P_i, and $F(P_1, ...P_k)$ is a proposition in these variables, and p is a number coding a proof in iFOL and $\text{Prov}(p, \ulcorner F \urcorner)$ is the metapredicate saying that the proof coded by p proves the formula coded by $\ulcorner F \urcorner$.

Kreisel defines *weak completeness* as the double negation of (1).

His main claim is that it is *implausible* that iFOL is complete, but it could be weakly complete. We know from Leivant's 1976 result that assuming RED, iFOL is not complete.

Here Kreisel makes a case for studying weak completeness. He cites a result of Gödel, labeled 2.

$$\forall \alpha\!:\!\mathcal{B}.\sim\sim \exists n\!:\!\mathbb{N}.A(n, \alpha) \Rightarrow \forall \alpha\!:\!\mathcal{B}.\exists n.A(n, \alpha) \tag{2}$$

Where $\mathcal{B}$ is the *full binary spread* with values in $\mathbb{B}$.

We will study there spreads later in terms of free choice sequences of truth values. We have added free choice sequences to Nuprl this year (2015) for reasons to be discussed later in the course.