Logic of Programs	**Lecture 12**
CS 6860 Fall 2015	**Thurs. Oct 1, 2015**

Lecture 12

In lecture 11 we were looking at how to construct proofs from evidence. We will eventually examine this in more detail and sketch the proof of completeness for iPC. Before going into this topic we will discuss the idea of *uniform evidence* and apply it to thinking about *admissible rules*, a topic with some interesting research questions that we might be able to answer using uniform evidence.

We will also continue our discussion of Brouwer's intuitionism and its contributions to logic, and lately to computer science. While mathematics has had a strong constructive aspect at least since Euclid and Algoritmi, that aspect did not add much to logic until Brouwer.

1. Uniform evidence.

   Consider the evidence term for $\sim\sim (P \vee \sim P)$.

   $$\lambda(f.\text{ap}(f;\text{inr}(\lambda(p.\text{ap}(f;\text{inl}(p))))))$$

   Notice that it does not depend on the proposition P. In the case of $(P \vee \sim P)$ we do not have constructive evidence. If we want to think about what it might be, we see potential terms such as $magic(P)$. $Magic$ might have the "power" to find evidence given access to proposition P, and produce the term inl($magic(P)$), or find evidence that evidence for P would lead to evidence for False and return inr($magic(P)$).

   Consider the evidence for $\sim A \vee \sim B \Rightarrow \sim (A \,\&\, B)$:

   $$\lambda(d.\lambda(ab.\text{spread}(ab;\ a, b.\text{decide}(d; na.\text{ap}(na; a); nb.\text{ap}(nb; b)))))$$

   Notice that it does not depend on the particular A, B. It is *uniform* in A and B. These could be **any** propositions.

2. Admissible Rules.

   The study of admissible rules is a small area of proof theory for intuitionistic and model logics. It seems that there is an opportunity to shed light on the results and themes using the computational semantics for iPC and iFOL. There is little known about the computational meaning of admissible rules.

   We looked at the example of Mint's rule:

   $$\vdash_{iPC} (A \Rightarrow B) \Rightarrow A \vee C \text{ implies } \vdash (A \Rightarrow B) \Rightarrow A \vee (A \Rightarrow B) \Rightarrow C.$$

   What is the definition of an admissible rule? Why is this not a topic in classical proof theory?

What can computation add?

What is an admissible rule in general?

3. Reading topic.

   The philosophy underlying Brouwer's intuitionism. How does it relate to constructivism in mathematics?

   There are separate notes on this topic related to van Atten's book, *On Brouwer*, that will be distributed in class. The book is no longer in print. You can read some of van Atten's writings at `http://plato.stanford.edu/entries/intuitionistic-logic-development/`.