Logic of Programs	**Lecture 10**
CS 6860 Fall 2015	**Thurs. Sept 24, 2015**

Lecture 10

Topics

- Reflecting on Evidence Semantics.
- New semantics.
- Further discussion of classical PC.

  Kolmogorov embedding into iPC.

  Gödel embedding into iPC.
- Preview of the squash/virtual evidence semantics for PC.

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1. Reflecting on Evidence Semantics

   We see both philosophical and technical reasons for exploring this new semantics. On the philosophical side we hear phrases such as "mental constructions" and intuition used to account for human knowledge.

   On the technical side we see that computers are important factors in the technology of knowledge creation.

   For PC we have a clear *computational semantics* for understanding the logical operators. Understanding the atomic propositions and their "evidence semantics" is more subtle. One way it will be clarified is by looking at first-order and higher-order logic and type theory. Those logics refine and explicate our notion or a proposition. Type theory also shows promise in providing a semantics for natural language (see Ranta's book *Type-theoretical Grammer* [1]. The semantic framework provided by type theory is a topic we will explore further.

2. New semantics for iPC

   We now allow the meaning of False in iPC to be either the empty type, Void, or a type with exceptions. This account has been formalized in Nuprl by Mark Bickford. We will examine it next week. He has proved this theorem using Nuprl.

   > **Theorem**. If formula F is provable in iPC, then it is uniformly valid in semantic structures that assign to False either an empty type (Void) or a type with exceptions.

   We will cover this topic next week, examining the Nuprl formal account.

3. Further discussion of classical PC.

   We look briefly again at the evidence for $P \vee (P \Rightarrow \text{False})$, i.e. $P \vee \sim P$. We noted that if we want to compute with this classical rule, then we need an oracle for P, which we called $magic(P)$. The only proof rule of iPC that needs an oracle is $P \vee \sim P$.

4. Brief discussion of the "squash" type and its logical value.

   $\{P\} = \{x\!:\!\text{Unit}|P\}$

   New rule: $\sim\sim P \Rightarrow \{P\}$.

   Compare to Kolmogorov's axiom $\sim\sim P \Rightarrow P$ for defining PC.

References

[1] Aarne Ranta. *Type-theoretical Grammar*. Oxford Science Publications. Clarendon Press, Oxford, England, 1994.