Logics of Programs	**Some Homework Problems**
CS686 Spring 1998	**Solutions Posted Tuesday, February 10**

1. An *accumulation point* of a set A is a point x such that every open set containing x has an infinite intersection with A. Show that in a compact space, every infinite set has an accumulation point. (The accumulation point itself need not be in A.)

2. A set A is *compact* if every open cover of A has a finite subcover. Show that every closed subset of a compact space is compact.

3. Show that the continuous image of a compact set is compact. (The continuous image of a set $A \subseteq X$ is the set $h(A) = \{h(x) \mid x \in A\} \subseteq Y$ for some continuous map $h : X \to Y$.)

4. Show that the continuous image of a compact space in a Hausdorff space is closed. That is, if $h : X \to Y$ is continuous, X is compact, and Y is Hausdorff, then $h(X)$ is closed.

5. (a) The *free Boolean algebra on generators* X is the set of propositional expressions over propositional symbols X modulo propositional equivalence (recall φ and ψ are *equivalent* iff $\varphi \leftrightarrow \psi$ is a tautology). For X a set of size n, describe the Stone dual of this Boolean algebra.

   (b) Suppose X is countably infinite. What is the Stone dual? (Hint. You have seen it in class.)

6. (a) Let B be a Boolean algebra. Show that a set $F \subseteq B$ is a filter iff there exists a Boolean algebra A and a homomorphism $h : B \to A$ such that $F = \{x \mid h(x) = 1\}$.

   (b) Show that F is an ultrafilter iff there exists a homomorphism $h : B \to \mathbf{2}$ such that $F = \{x \mid h(x) = 1\}$, where $\mathbf{2}$ is the two-element Boolean algebra.