Logics of Programs	**Homework 1**
CS686 Spring 2001	**Solutions Posted Wednesday 1/31**

1. For ordinals α, β, define $\alpha < \beta$ if $\alpha \in \beta$, $\alpha \leq \beta$ if $\alpha < \beta$ or $\alpha = \beta$.

   (a) Define a partial order on ordinals by $(\alpha, \beta) \leq (\alpha', \beta')$ if $\alpha \leq \alpha'$ and $\beta \leq \beta'$. Show that $\leq$ (or more accurately, the strict partial order associated with $\leq$) is well-founded. Use the definition of ordinal on p. 14 of HKT.

   (b) Prove that the ordinals are totally ordered: for all α, β, either $\alpha < \beta$, $\alpha = \beta$, or $\beta < \alpha$. (*Hint.* Well-founded induction on the relation in (a).)

2. (a) Exercise 1.21 p. 25 in HKT: a countable union of countable sets is countable.

   (b) Prove that the ordinals $\mathrm{ord}_T(\varepsilon)$ labeling roots of well-founded trees are exactly the countable successor ordinals. See HKT pp. 50–51 for definitions.

3. HKT Exercise 1.17 p. 24.

4. HKT Exercise 1.18 p. 25.