CS 6840 Algorithmic Game Theory April 13, 2020

Lecture 25: Simultaneous Auction for Unit-Demand Bidders

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Introduction

Last week we covered single item auctions. This week we are going to cover simultaneous auction for multiple items. Suppose we have items $1, \dots, m$ and bidders $1, \dots, n$. All players want at most one item and the value for player i for item j is v_{ij}.
If player i gets a set $A \subseteq \{1, \dots, m\}$ then we say that the value for this is the max item value. That is:

$$v_i(A) = \max_{j \in A} v_{ij} \tag{1}$$

this is called *free disposal*. Note that all items are different, but each player only wants one. For instance, consider advertisements on a web page where we want to advertise car companies. Different car companies have different beliefs for placement of ads v_{ij} but there may be a rule that you cannot put multiple ads of the same car company on the same page.

How do we maximize social welfare? Given v_{ij} values, how do we maximize social welfare? Given that the set of items A_i goes to person i, then:

$$SW(A) = \sum_i v_i(A_i) \tag{2}$$

Since we must allocate at most one item to each person, we can think of this as a maximum value bipartite matching problem. In particular, the left set of vertices denote each person i, and the left side denotes each item j. The edge between a player i and item j is v_{ij}. Now the maximum value bipartite becomes a problem to maximize the perfect matching. This is our ideal scenario, we hope that we do well compared to this.

Auction System

Today we will look at simultaneous first price. So each player i submits a bid b_{ij} for all j. Next, we will run **first price auction** for all items. What's a strategy to bid on this auction?

- High bid on one item only since we don't want to pay a lot for a second item (e.g. bid on one item only)
- Low bids on all but one item (hedge your bets)

Today, given a Nash equilibrium of this game, how good is social welfare?

Aside: Bayesian vs Non Bayesian The non-Bayesian version of Nash is that other values are fixed. You may know other's values, but more importantly, you *know what others are using as bids and don't want to change*. In particular, to be Nash, even *if you know what others are doing, you don't want to change*. In a Bayesian version, you sort of know what they are doing using expectations, but you don't want to change.

Using Smoothing Frameworks

If we have a "recommended bid" called $b_i^* \quad \forall i$ which they all don't have to follow, but say we know the recommended bid at Nash. Then we know that the utility we get is:

$$u_i(b) \geq u_i(b_i^*, b_{-i}) \tag{3}$$

whatever we are getting is greater than the recommended bid at Nash.

By smoothness (magic):

$$\sum_i u_i(b_i^*, b_{-i}) \geq \frac{1}{2}OPT - REV(b) \tag{4}$$

where OPT is the maximum possible social welfare and $Rev(b)$ is the revenue to auctioneer at bids b. If we have this inequality, then (3) + (4) is Nash implies that $SW(b) \geq \frac{1}{2}OPT$:

$$\sum_i u_i(b) \geq \sum_i u_i(b_i^*, b_{-i}) \geq \frac{1}{2}OPT - Rev(b) \tag{5}$$

where the first inequality is from Nash and second from smoothness. Recall that $SW(b) = \sum_i u_i(b) + Rev(b)$.

Next, defining b_i^* and prove:

$$\sum u_i(b_i^*, b_{-i}) \geq \frac{1}{2}OPT - Rev(b) \tag{6}$$

Now we define: $b_{ij}^* = v_{ij}/2$ if i wins j in OPT, and $b_{ij}^* = 0$ otherwise. Note that b_{ij}^* depends on OPT, which in turn, depends on other people's values. So player i may not know b_i^* but Nash implies that they do not regret it.

Now we need to prove smoothness. Note that $u_i(b_i^*, b_{-i}) \geq 0$ is true for all i. If i gets j in opt, then we want something stronger. We consider two cases: if b_i^* makes i win item j

$$u_i(b_i^*, b_{-i}) \geq \frac{1}{2}v_{ij}$$

and if b_i^* does not make i win item j:

$$u_i(b_i^*, b_{-i}) \geq \frac{1}{2}v_{ij} - \max_k b_{kj}$$

so someone else won it, so they are paying higher. Now we add up all players.

$$\sum_i u_i(b_i^*, b_{-i}) \geq \sum_{(i,j)\in M^*} \frac{1}{2}v_{ij} - \sum_j \max_k b_{kj}$$

The first summation on the RHS is exactly $\frac{1}{2}OPT$ and the second summation is $Rev(b)$

Summary

So we talked about simultaneous first price, discussed how bidding is a bit tricky here and we formulated social welfare, and we used the smoothness framework with an a priori b_i^* value which is that imagine what the optimal is, and bid half on that. Next, think about what did we prove? the notation we have used have assumed that this is a deterministic equilibrium. In the next class, does this work on probabilistic or mixed nashes? How about Bayesian and learning? Today we focused on b that were pure strategy Nash, and the question we pose is how far does this go? What about mixed learning and Bayesian?