CS6840 - Algorithmic Game Theory (2 pages) Spring 2014

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"Second prize" with one item was truthful and thus too simple. An application of Generalized Second Prize auctions is in found in selling ads next to search.

Simple model: advertisers bid on ads

$b_i \rightarrow$ willingness of advertiser i to pay for a click (bidding language allows dependence on lots of info)
[Budget$B_i = \max$ total "over a day"] we ignore today $\rightarrow$ think of it as so big that we won't reach it.

model advertiser's value: v_i as value per click (depends on search term, time of day, location of search etc...), 0 for no click
Questionable assumption: is the value really 0 if the advertiser's ad was displayed?

Probability of getting a click

position j for ads $\rightarrow$ has probability α_j to get a click
ad i itself has probability γ_i for getting a click (depends like v_i on everything)
Questionable assumption: ad i in position j gets click with probability $\alpha_j\gamma_i$

Optimal assignment

The value of advertisement i in position j is $v_{ij} = v_i\gamma_i\alpha_j = v_i\mathbb{P}[i \text{ gets clicked on in position } j]$

We may assume, after renumbering, that $\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n$ and $v_1\gamma_1 \geq v_2\gamma_2 \geq ... \geq v_n\gamma_n$. The optimal assignment is then given by assigning ad i to α_i (this can be seen with a simple exchange-argument: if an assignment is not sorted like this, then there is some pair $i, i+1$ sorted in the wrong order. Swapping them will increase $\sum_i v_i\mathbb{P}[i \text{ gets clicked on}]$).

This gives rise to the following algorithm:
<u>ALG:</u>
ask bidders for b_i
compute γ_i
sort by $b_i\gamma_i$
assign slots in this order.

Pricing

Historically speaking there have been the following versions:
Version 1 (First Price): Pay b_i if clicked. Problem: consider two players bidding for two advertisement locations. for a while they keep outbidding each other for the better advertisement location until eventually, one decides to take the worse one for very little - but then the other one can take the better one for just a little more and the outbidding starts all over again $\rightarrow$ unstable.
Version 2: set p_i to be the minimum needed for i to keep her slot, i.e.: $p_i = \min\{p : p\gamma_i \geq b_{i+1}\gamma_{i+1}\} = \frac{b_{i+1}\gamma_{i+1}}{\gamma_i}$.

Observation: $p_i \leq b_i$. Is this truthful?

Consider two players, $v_1 = 8, v_2 = 5, \alpha_1 = 1, \alpha_2 = .6, \gamma_1 = \gamma_2 = 1$. If both players bid truthfully, player 2 pays 0, but player has value $(v_1 - p_1)\alpha_1 = 3$ (her expected utility), but with an alternate bid - say 4 - $(v_1 - 0) = 8 \cdot .6 = 4.8 > 3$, so the mechanism is not truthful!

Next class:smoothness-style analysis of a Price of Anarchy result for generalized 2nd-price (assumption: $b_i \leq v_i \forall i$ - How bad is this assumption?)