CS 6840 – Algorithmic Game Theory (3 pages) Spring 2012

Lecture 1 Scribe Notes

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# 1 Lecture 1 – Monday 23 January 2012 - Mostly Intro

## 1.1 Overview

Special - possible extra credit for improving wikipedia articles on topics related to this course.

Motivating connecting Game Theory and Computer Science.

- The Internet. Computer Science used to be summarized by trying to make computers effective. In comparison, today, there is the Internet connecting everything and the idea that one person designing one algorithm controls everything is just wrong.
- Mechanism design. Really an engineering discipline, designing a game to get a desired outcome.

Different perspectives. Ways in which our perspective is different from the traditional approaches to game theory.

- We care about algorithms and as mentioned previously we know how to design and analyse them. Economists come from a different historical perspective when it comes to approaching these problems and there isn't the same emphasis on the complexity of finding the equilibria of a system. From the Computer Science perspective, taking up this approach brings us to interesting complexity results which we'll explore throughout the semester.
- Simplicity of mechanisms. Particularly coming from a systems background, mechanisms must be simple.
- (Approximate) optimality. Say average response or delivery time. Often times, our objective functions don't have such clear cut measures of utility. Time can be relative. Do we care about the difference between 7 and 8 seconds?

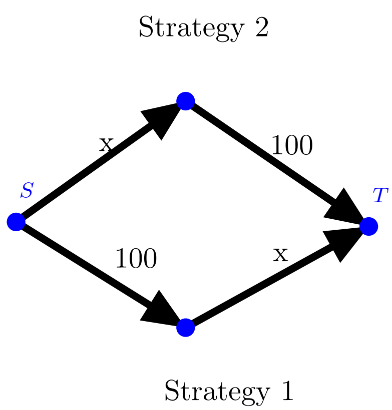
See the web page for a rough syllabus for the course.

Since the book, mechanism design has changed a bit. We will be using Jason Hartline's book on this topic more than the book listed as the text for this course.

Ken Binmoore basic introduction to game theory is a good, quick intro to game theory.

## 1.2 An Example – Braess' paradox

We'll start by talking about Braess' paradox. Consider the following graph.



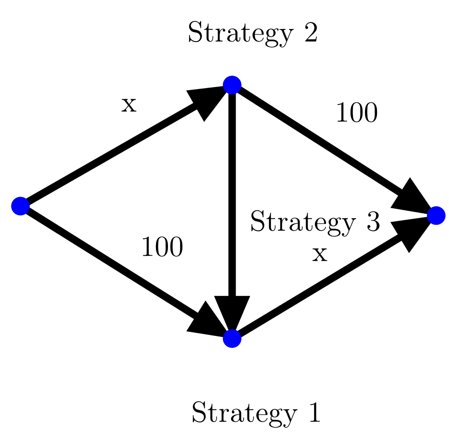

For every edge there is $d_e(x)$ = delay on $e$ if $x$ players use this edge. We will assume $d_e(x)$ is a monotone increasing (i.e. non-decreasing) function. Here strategy 1 is the bottom path and strategy 2 is the top path.

Notice the two edges with weight 100. These edges take 100 seconds to cross regardless of congestion, whereas the edges labelled $x$ are congestion sensitive.

Our goal: predict what will happen here. The obvious prediction is that the players should split half-half. So 50 choose strategy one and another 50 choose strategy two. This prediction is a *(pure) Nash equilibirium* - an outcome so that everyone chooses a strategy such that deviating from their strategy will not improve their outcome. Total time is 150 for all players. If someone following one strategy tries to deviate, they will not improve their outcome. In this example, someone switching from strategy one to strategy two will increase their travel time by a second worsening their outcome.

Now we add an extra edge with no delay and see how this affects the strategies. Adding the extra edge provides an additional strategy: strategy 3 where a player starts out using the top edge, then uses the edge we just added and then takes the last edge to the sink.

Start with the 50-50 solution, but this is no longer an equilibrium. Now we claim that all 100 choosing strategy 3 is a Nash equilibrium with total delay 200 for all players.




To see this, notice that a player taking strategy 1 or strategy 2, then changes his delay time from 200 to 200 so no improvement, thus we have the desired equilibrium.

This equilibrium is not unique. 99 could choose strategy 3 and 1 could choose strategy 1. This would still be at equilibrium. The lonely guy might be jealous of his 99 friends for getting delay 199, but changing his strategy would not help his outcome from 200 so the system is still in equilibrium. There are many more similar such equilibria.

Claim that at least 98 players must choose strategy 3 and at most 1 player can choose strategy 1 and at most 1 player can choose strategy 2 for Nash equilibrium.

Contrasting the dynamics of this game with the previous, is that now total delay is 200 for most players and at best 198 for some players. In either case, making life worse for all characters. Hence Braess' paradox – adding an edge to the graph made life worse for all the players.

Questions we will look at: how do you find these equilibria? Why do we get paradoxes like this one? Are situations like this paradox really so bad? Adding the edge didn't make life that much worse for players. Can players find these equilibria?

We will look at various ways for looking at how to prevent paradoxes like this one. We will heavily rely on some sort of 'rationality' for the players. Later we'll go back and look at a more intricate definition of equilibrium. Our current definition is good enough when these equilibria are fairly stable and unique. There are however criticisms of Nash Equilibrium and next time we'll get some examples of where Nash Equilibrium doesn't capture everything we want.

Next time we'll consider the significance of various objective functions. Maybe you don't just want to make lots of money – maybe it's more important to make more money than your neighbors? Or maybe you don't just want to make as much money as possible in a game, but rather to make sure you win the game. We will explore trade offs between various measures of utility.