1 Lecture 30 – Monday 02 April 2012 - A Common Framework

1.1 Overview of next two lectures
• checking how far we’ve gotten to greedy and item auction goals
• put two auction types into common framework, so we can make general statements

1.2 Recap of 2 Auctions
Both Greedy and Item
• are combinatorial auctions
• $S =$ set of items
• $v_i(A) \geq 0$ value for set $A$, user $i$
• assume free disposal: $v_i(A) \geq v_i(B)$ if $A \supseteq B$
where the latter two assumptions are the only assumptions we have on value so far.

1.2.1 Greedy (Wednesday) - from Lucier-Borodin paper
• bid: sets, and bids $b_i(A)$ (i.e., not all sets need bids)
• select max $\frac{b_i(A)}{\sqrt{|A|}}$ among sets $A$ still available
• critical value pricing (i.e. smallest bid that would have won item; natural analog to second price)

1.2.2 Item-Auction (Friday)
• bid: bid per item $= b_i(a) \geq 0$
• for each item, run first or second price to determine winner and price

1.3 $(\lambda, \mu)$ Framework
1.3.1 Greedy
From original analysis, we have
1. If $\Theta_i(A)$ is critical bid for set $A$, then

$$v_i(O) - \Theta_i(O) \leq v_i(A_i)$$

if $A_i$ is allocated by algorithm for all sets $O$ (in a Nash). In other words, player $i$ could just bid for set $O$.

2. If algorithm is a $c$-approximation, then

$$\sum_i \Theta_i(O_i) \leq c \sum_i b_i(A_i)$$

if $A_i$ is algorithm’s allocation and $O_i$ is optimal.

3. Assumes bidders are conservative: $b_i(A) \leq v_i(A)$.

**Claim:** This is a $(\lambda, \mu)$-smoothness proof. Recap of smoothness:

Smoothness had cost and utility versions. This is a utility version. We have strategy $s^*$ that leads to solution maximizing $\sum_i U_i(s^*)$, where $s^* = (s^*_1, ..., s^*_n)$ and $U_i(s^*)$ is utility of player $i$ in resulting outcome.

If, for all $s$,

$$\sum_i U_i(s^*_i, s_{-i}) \geq \lambda \sum_i U_i(s^*) - \mu \sum_i U_i(s)$$

then game is $(\lambda, \mu)$-smooth. This implies

1. if $s$ is Nash, and $s^*$ is opt, then

$$\sum_i U_i(s) \geq \frac{\lambda}{1 + \mu} \sum_i U_i(s^*)$$

(i.e., price of anarchy bound), and

2. if all players have no regret $s^1, ..., s^T$, then

$$\frac{1}{T} \sum_{t=1}^T \sum_i U_i(s^*) \geq \frac{\lambda}{1 + \mu} \sum_i U_i(s^*)$$

So can we convert greedy auction bound to $(\lambda, \mu)$-smoothness bound form?

- idea: one optimal strategy $s^*$ for player $i$ is to bid for $O_i$ only; $b_i(O_i) = v_i(O_i)$
  - critical value = 0 since sets disjoint
  - this is as good as possible
- so take $\lambda = 1, \mu = c$
Issue: How do we evaluate $\sum_i U_i(s^*_i, s_{-i})$?

$$\sum_i U_i(s^*_i, s_{-i}) = \sum_i v_i(O_i) - \sum_i \Theta_i(O_i) \geq \sum_i v_i(O_i) - c \sum_i v_i(A_i)$$
due to $c$-approximation and conservation assumption.

1.3.2 Item Auctions

Interjections:

- Fire Drill!!. 15 minute loss.
- Note: we never even proved Nash exists, and/or if we can find it. But we do know learning exists and has nice properties.

Note that first-price doesn’t fit into $(\lambda, \mu)$ framework. Second price almost does:

Suppose $O_1, ..., O_n$ is an optimal allocation. Then

- for each item $a$, $p(a) = \max_i \{b_i(a)\}$
- if $v_i(O_i) \geq \sum_{a \in O_i} p(a)$, then bids $b_i = p(a) + \epsilon$ for all $a \in O_i$.

At Nash,

$$v_i(A_i) \geq v_i(O_i) - \sum_{a \in O_i} p(a)$$

(otherwise they would have bid for $O_i$). But then

$$\sum_i v_i(A_i) \geq \sum_i [v_i(O_i) - \sum_{a \in O_i} p(a)]$$

$$= \sum_i v_i(O_i) - \sum_a p(a)$$

$$= \sum_i v_i(O_i) - \sum_a \sum_{a \in A_i} p(a)$$

$$\geq \sum_i v_i(O_i) - \sum_i v_i(A_i)$$

So this is almost smoothness with $\lambda = 1$, $\mu = 1$ (the right side is good). But there is a problem: No clear bidding strategy for $O_i$!. Remember we found bids by adding $\epsilon$ to price in current solution; in other words, solution $s^*$ is not independent of $s$. More next time.