Algorithmic Game Theory	Problem Set 2
CS 684 Spring 2008	Due Monday, March 8th

We will maintain a FAQ for the problem set on the course Web page. You may use any fact we proved in class without proving the proof or reference, and may read the relevant chapters of the book. However, you may not use other published papers, or the Web to find your answer.

A full solution for each problem includes proving that your answer is correct. Please start by explaining what is the high-level idea of the solution (the main insights/nontrivial things necessaries to solve them problem). If you think its useful you may add also pseudocode for details. Do not submit a code only. It can make the solutions more readable if you introduce convenient notation and use it.

If you cannot solve a problem, write down how far you got, and why are you stuck. You may solve the problem with a partner. Please hand in a shared problem set with both your names on the solutions. We are happy to help finding partners, let us (Renato or Hu) know if you don't have a partner, and would like one.

Solutions can be submitted on CMS, or handed in at Renato or Hu's office. Please type your solution, to make it easier to read.

(1) In Problem set 1, we asked (Problem 4d) about the asymmetric grouping game (i.e., when u_{ij} may not equal u_{ji}). Prove that the in general the asymmetric grouping game is not a potential game. If you proved this as part of your solution to problem 4d, either resubmit the solution, or let us know to read it. If you proved only that the social welfare is not a good potential function for this game, now prove that the game has no potential function.

(2) In the bicriteria bound we proved on February 10th we assumed that all flow is very sensitive to delay, and the flow is in a real Nash equilibrium. It is maybe more reasonable to model a stable state as one of the approximate equilibria, i.e., assume only that for all pairs of paths P and Q connecting the same pair of terminals, if $f_P > 0$ then $\ell_P(f) \leq (1+\varepsilon)\ell_Q(f)$ for some sensitivity parameter $\varepsilon > 0$. Show that there is a version of the bicriteria bound that holds also for all approximate Nash flows.

(3) Solve problem 1.7 in the book.

(4) Show that in the proportional sharing mechanism of on a single link (of Feb 15-17), on any monotone increasing and nonnegative utilities, a Nash equilibria must always allocate non-zero amount of the resource to at least two users.

(5) Consider the local network formation game of Section 19.2 (discussed in class on Feb 24). Recall that in this game players are nodes, and each player chooses a subset of its neighbors to build an edge to it. The cost of α for every edge built, plus a cost derived by the pairwise distances in the resulting undirected graph.

(a.) Show that there is an n_0 so that a path of length $n \geq n_0$ is not an equilibrium in this game for any value of α. (Note that n_0 is a number, may not depend on α.)

An alternate variant of this game commonly used in the literature is when the edge between a pair of nodes (u, v) has to be paid by both players (charging $\alpha/2$ to each). In this case, we will need two players to add any edge, so we consider a variant of Nash, when two player coalitions can coordinate a move together (player v and w can together drop any edges they currently pay for, and possibly buy the edge connecting them, but only if this move benefits **both** v and w). We will call such a solution "pairwise stable"

(b.) Show that for every $n > 0$, there is an $\alpha > 0$, so that the path of length n is a pairwise stable solution for the game.

(6) Solve problem 19.17 in the book.