CS 684 Algorithmic Game Theory	Scribe: Mauro Sozio
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1 Two persons game

In this lesson we will talk about the two persons game and in particular, about the zero-sum games. In a 2 persons game there are one or more matrices which describe the game.

For example in the rock-paper-scissors game there are two players the row player and the column player. A matrix A specifies the payoffs for the row player. So, if the row player chooses row i and column player choose column j the former gives to the latter a_{ij}. In Table 1 we report the matrix with the payoffs.

Another example of a 2 persons game is the cooperate-defect game. In this game we have two matrices. One matrix A for the payoffs of the row player and another matrix B for the column player. If the 2 players cooperate then they will pay a low payoff but if one of the player cooperates and the other defects then the former will pay 0 and the latter will pay the maximum payoff. Finally, if both defect they will pay a medium payoff. In Table 2 we report the two matrices.

We say that the rock-paper-scissors-game is a zero-sum games because $A + B = 0$. In general every game in which the looser pay the winner is a zero-sum game. The cooperate-defect game is an example of a non-zero sum game.

2 Randomized Nash

A randomized Nash is a probability distribution p of rows and q of columns, where p is a row vector and q is a column vector. For example the rock-paper-scissors game has a unique randomized Nash and it is a uniform distribution on rows and columns.

Given the probability distribution p and q let us compute the expected benefits for the two players. We have that:

$$\textit{expected benefit for row player} = pAq$$

$$\textit{expected benefit for column player} = pBq$$

The probability distributions p and q are in a Nash equilibrium if and only if given q row player does not want to change p and given p the column player does not to change q. That is, if $p_i > 0$ the ith entry of Aq is the entry with the maximum value and $q_j > 0$ implies that jth entry of pB is the entry with maximum value.

	R	P	S
R	0	1	-1
P	-1	0	1
S	1	-1	0

Table 1: The payoffs of the row player in the rock-paper-scissors game.

	C	D
C	1,1	5,0
D	0,5	4,4

Table 2: The payoffs of the two players in the cooperate-defect game.

What does it mean for a zero-sum game to be in a Nash equilibrium? Remember that in this case we have only a matrix A that contains the payoffs of the row player and the win of column player. So from the row perspective Aq is the expected loss on rows, and from the column perspective pA is the expected win on columns.

Suppose row publishes his strategy, what is the minimum guaranteeable loss for row? Let v_R this value. Row player wants to make maximum entry in pA as small as possible, so he wants to minimize v_R such that $pA \leq v_R e \quad e = (1, \ldots, 1)$.

And what about the maximum guaranteeable win for column player when he publishes his strategy? Let v_C this value. He wants to make the minimum entry of Aq as small as possible, so he wants to maximize v_C such that $Aq \geq v_C e \quad e = (1, \ldots, 1)$ where e is a column vector.

What is the relation between v_R and v_C? In the next class we will prove that $v_R = v_C$ on all zero-sum games.