CS 683 Advanced Design and Analysis of Algorithms April 21, 2008

Lecture

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Belief Propagation (Part II)

When the factor graph is not a tree, we can start with a random assignment and repeatedly update it. Furthermore, we are guaranteed convergence in cases where the graph has only a single loop.

Proof: Consider the following factor graph:

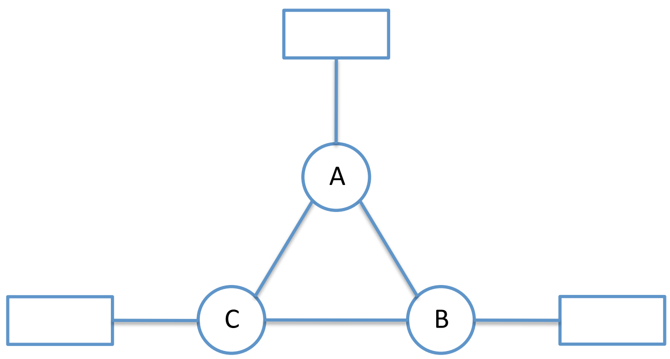

Figure 1: A factor graph with a single loop.

We can denote the MAP probability as:

$$\begin{aligned} P(A,B,C) &= \frac{1}{\mathcal{Z}}\Psi(A,B)\Psi(B,C)\Psi(A,C) \\ &= e^{\log P(A,B,C)} \\ &= e^{\overbrace{\log\Psi(A,B)+\log\Psi(B,C)+\log\Psi(A,C)}^{J}} \end{aligned}$$

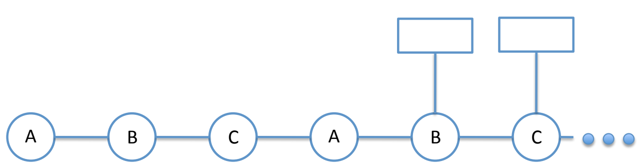

Figure 2: An unwrapped version of the factor graph in Figure 1.

But since we're maximizing, we can instead consider just the exponent (J). Now shifting our attention to the unwrapped version of the factor graph, as in Figure 2 above, what we have is:

$$\max J_n = \max\ (nJ + J')$$

where J' is the portion of the tree not stabilized, and n corresponds to the number of times the loop has been unwrapped. Since we know J_n is maximized (it's a tree), it must also be that J is maximized [1].

References

[1] Yair Weiss. Correctness of local probability propagation in graphical models with loops. *Neural Computation*, 12(1):1–41, 2000.