CS 683: Advanced Design & Analysis of Algorithms February 11, 2008

Lecture 10

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1 Introduction

Algorithms

- Don't ask complexity
- Don't ask correctness
- Ask them if they are useful
- Achieve communication at Shannon limit

Examples

- Belief Propogation in Bayesian Networks
- Viterbi Algorithm (PDAs)
- Kalman Filter (Airplanes)
- Turbo Algorithm - first to meet Shannon limit

Outline

- Problem Statement
- Graphical Models
  - Bayesian Networks
  - Factor Graphs
  - Pairwise Markov Random Fields
- Equivalence of Graphical Models

Reference: Yedidia, Freeman, Weiss. "Understanding Belief Propagation and it's generalization".

2 Problem Statement

- Discrete system, state vector $\vec{x}$, $x_i \in Dom$
- Weight of each state (configuration), $weight(\vec{x}) \in \mathbb{R}_0^+$
- Represent the weight concisely $\Rightarrow$ Graphical Models
  - Graph (directed or undirected)
    $V \sim$ state variables
    $E \sim$ (in)dependencies

– Weights are "distributed" among the nodes and edges

Special Case
Configurations ($\vec{x}$) $\sim$ possible worlds in some problem space
$weight(\vec{x}) = P[\vec{X} = \vec{x}]$, where P is the probability of some event occuring.

- Joint Probability $P[\vec{X} = \vec{x}]$
- Marginal Probability $P[X_i = x_i] = \sum_{\vec{x}_{-i}} P[\vec{X} = \vec{x}]$, where $\vec{x}_i \triangleq (x_j)_{j \neq i}$

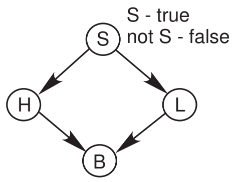
Questions (Tasks/Queries)

- Compute the marginal probabilities
- Compute the most likely configuration $\underset{\vec{x}}{\operatorname{argmax}}\, weight(\vec{x})$

Inference is the mechanism to answer queries (#P-complete problems)

3 Bayesian Networks

- Directed Acyclic Graph (DAG)
- V = state variables
- Independence Property: X_i is independent of Non-descendants(X_i) given Parents(X_i)

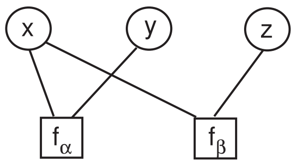


$$\begin{aligned} P(s,h,l,b) &= P(b|s,h,l)\ P(s,h,l) \\ &= P(b|h,l)\ P(h,l|s)\ P(s) \\ &= P(b|h,l)\ P(h|s)\ P(l|s)\ P(s) \\ \Rightarrow P[\vec{X} = \vec{x}] &= \prod_i P[X_i|Par(X_i)] \end{aligned}$$

- In Bayesian Networks, we may compute the probability of a given node given some evidence $P[X_i = x_i|evidence]$

4 Factor Graphs

- Undirected bipartite graph
- Two kinds of nodes:
  1. Variables $\bigcirc$ $x_i \in Dom$
  2. Factors $\square$ $f_\alpha(\vec{x}_\alpha) \in \mathbb{R}_0^+$, where $\vec{x}_\alpha$ is a subvector of $\vec{x}$
- $weight(\vec{x}) \triangleq \prod_\alpha f_\alpha(\vec{x}_\alpha)$



- SAT: $(x \vee y) \wedge (\neg x \vee z)$
  Let $\alpha = (x \vee y)$ and $\beta = (\neg x \vee z)$

$$f_\alpha = \begin{cases} 1 & \text{iff } (x \vee y) \\ 0 & \text{otherwise} \end{cases}$$

$$f_\beta = \begin{cases} 1 & \text{iff } (\neg x \vee z) \\ 0 & \text{otherwise} \end{cases}$$

- Probalistic Interpretation

$$\begin{aligned} P[\vec{X} = \vec{x}] &\propto weight(\vec{x}) \\ &= \frac{1}{z} weight(\vec{x}), \text{ where } z = \sum_{\vec{x}} weight(\vec{x}) \end{aligned}$$