

1 VC-dimension

A set system (x, S) consists of a set x along with a collection of subsets of x . A subset containing $A \subseteq x$ is shattered by S if each subset of A can be expressed as the intersection of A with a subset in S .

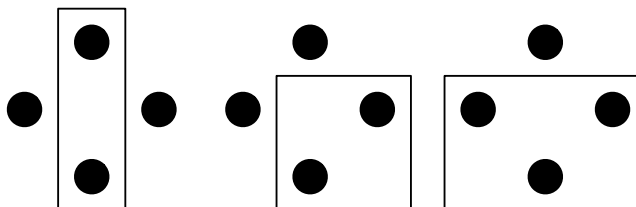
VC-dimension of a set system is the cardinality of the largest subset of A that can be shattered.

1.1 Rectangles

Let's try rectangles with horizontal and vertical edges. In order to show that the VC dimension is 4 (in this case), we need to show two things:

1. There exist 4 points that can be shattered.

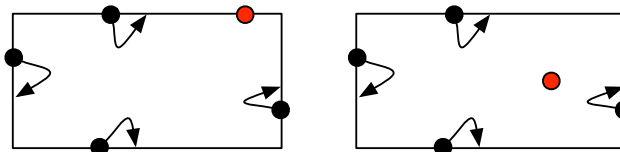
It's clear that capturing just 1 point and all 4 points are both trivial. The figure below shows how we can capture 2 points and 3 points.



So, yes, there exists an arrangement of 4 points that can be shattered.

2. No set of 5 points can be shattered.

Suppose we have 5 points. A shattering must allow us to select all 5 points and allow us to select 4 points without the 5th.



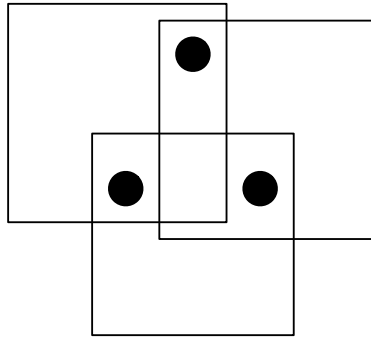
Our minimum enclosing rectangle that allows us to select all five points is defined by only four points – one for each edge. So, it is clear that the fifth point must lie either on an edge or on the inside of the rectangle. This prevents us from selecting four points without the fifth.

1.2 Squares

Let's try squares with horizontal and vertical edges. Again, we perform the two steps to show that the VC dimension is 3.

1. There exist 3 points that can be shattered.

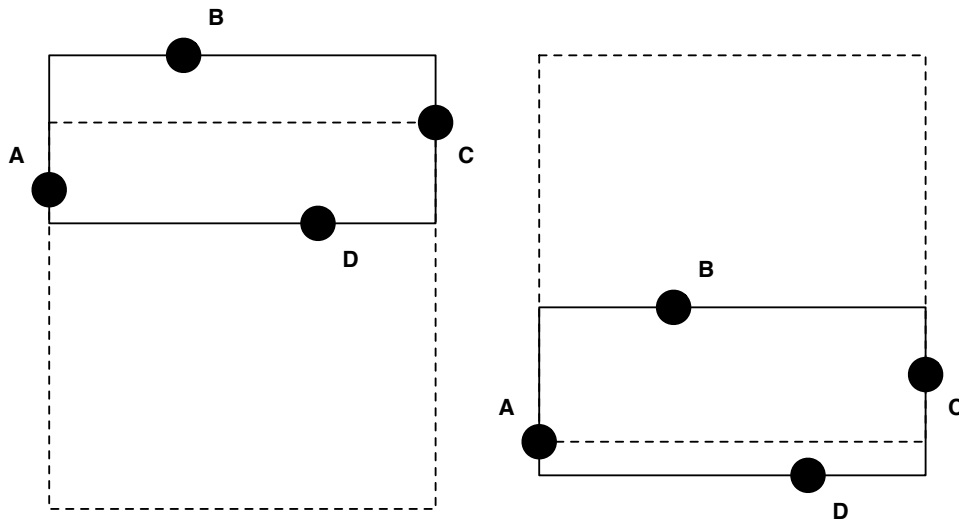
Again, 1 point and 3 points are trivial. The figure below shows how we can capture 2 points.



So, yes, there exists an arrangement of 3 points that can be shattered.

2. No set of 4 points can be shattered.

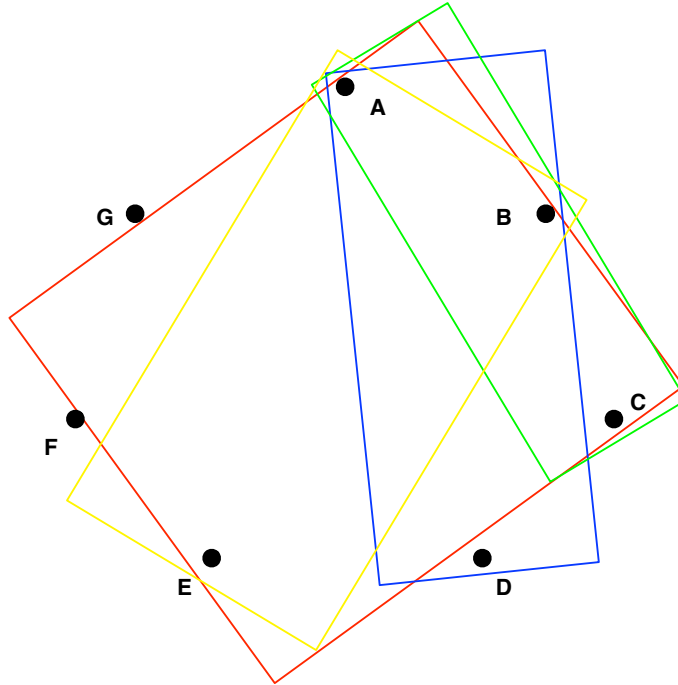
Suppose we have four points arranged such that they define a rectangle. Now, suppose we want to select two points (A & C , in this case).



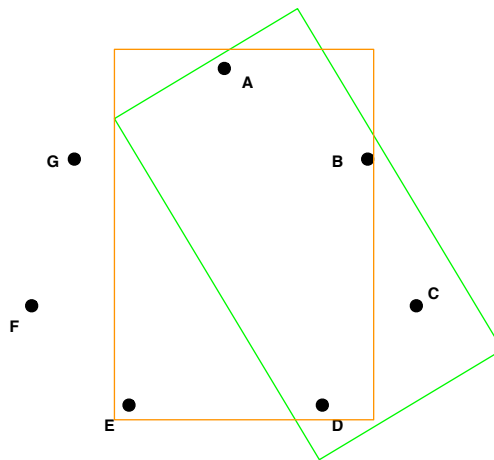
The minimum enclosing square for A & C must contain either B or D – so we can't capture just two points with a square.

1.3 Rotatable Rectangles

Rotatable rectangles are VC dimension 7. Suppose we have a regular heptagon. It's easy to get any subset of 0, 1, 2, 6, or 7 points. For 3 points, all configurations are symmetric to ABC , ABD , ABE , or ACE . The figure below shows how we can rotate rectangles to capture these 3 point configurations.



Similarly, we can capture the 4 point configurations.



1.4 Intervals on a Real Line

Suppose we want to shatter the real line with intervals. One interval gives us VC dimension 2.



Pairs of intervals give us VC dimension 4.

