

2. Let $A : V^2 \rightarrow \{0, 1, \perp\}$ be a matrix saying which edges have been queried and the answer, 0 meaning the edge is not there, 1 meaning it is there, and \perp meaning not queried yet.

At any stage, let $N = \{u \mid \exists v A(u, v) = 1\}$ and $I = V \setminus N$. As far as the algorithm knows at that point, N is the set of nodes that cannot be isolated and I is the set of nodes that could still be isolated.

At any intermediate stage, if the algorithm still must query all edges, then it had better be the case that for all $u \in N$ and $v \in N$, $A(u, v) \neq \perp$; that is, the edge (u, v) has already been queried and determined to be 0 or 1. Why? Suppose not. The algorithm already knows that neither u nor v can be isolated, so querying (u, v) would give no new information.

Similarly, at any intermediate stage, it had better be the case that for any $u \in I$, there must still be a node $v \in V$ such that $A(u, v) = \perp$, otherwise the algorithm already knows that u is isolated. But even stronger, there must still be a node $v \in N$ such that $A(u, v) = \perp$. Why?