Design and Analysis of Algorithms **Homework 4 Hints**
CS681 Fall 2005 **Saturday, October 1, 2005**

3. I mentioned that the set F of feasible flows on G is a closed, bounded, convex polyhedron in $\binom{n}{2}$-dimensional Euclidean space. Each flow f is a point in $\binom{n}{2}$-dimensional space, since it can be represented by a vector of real values $f(u, v)$, one for each $u, v \in V$, $u \neq v$.

   The set F is a *polyhedron*, since the constraints determining what is and is not a flow are all linear constraints. It is *convex* in the sense that if f and g are flows, then for any $\alpha \in \mathbb{R}$, $0 \leq \alpha \leq 1$, the function $\alpha f + (1 - \alpha)g$ is also a flow; that is, any point on a straight line drawn between f and g is also a flow. It is *bounded* in the sense that the flow values cannot be arbitrarily large or arbitrarily small due to the upper and lower bounds imposed by the capacity constraints: for all u, v, $-c(v, u) \leq f(u, v) \leq c(u, v)$. It is *closed* in the sense that it contains all its limit points; that is, if $f_0, f_1, \ldots$ is a sequence of flows that converge to a function $f = \lim_n f_n$, then f is also a flow.

4. Make a flow problem by adding a source s and sink t to G and new edges (s, u), (u, t) for all $u \neq s, t$. For each edge $(u, v) \in A$, set $c(v, u) = -1$ and $c(u, v) = N$, where N is some very large number. For all other edges (u, v), set $c(v, u) = 0$ and $c(u, v) = N$. For (u, v) not an edge, set $c(u, v) = c(v, u) = 0$. The capacity $c(v, u) = -1$ for $(u, v) \in A$ is effectively a lower bound of 1 on $f(u, v)$. Prove a lemma relating the number of directed paths covering A to the value of an integral s, t-flow. Then prove a lemma relating the number of elements of A no two of which lie on the same directed path to the value of a t, s-cut. Finally, use the max-flow min-cut theorem to relate these two things.