CS 681 Fall 1999 – Problem Set 4

Due in class Friday, November 12

(1) For functions $g_1, \ldots, g_\ell$, we define the function $\max(g_1, \ldots, g_\ell)$ via

$$[\max(g_1, \ldots, g_\ell)](x) = \max(g_1(x), \ldots, g_\ell(x)).$$

Consider the following problem. You are given n piecewise linear, continuous functions $f_1, \ldots, f_n$ defined over the interval $[0, t]$ for some integer t. You are also given an integer B. You want to decide: do there exist k of the functions $f_{i_1}, \ldots, f_{i_k}$ so that

$$\int_0^t [\max(f_{i_1}, \ldots, f_{i_k})](x)\ dx \geq B?$$

Prove that this problem is NP-complete.

(2) Given a set of finite binary strings $S = \{s_1, \ldots, s_k\}$, we say that a string u is a *concatenation* over S if it is equal to $s_{i_1} s_{i_2} \cdots s_{i_t}$ for some indices $i_1, \ldots, i_t \in \{1, \ldots, k\}$.

A friend of yours is considering the following problem: Given two sets of finite binary strings, $A = \{a_1, \ldots, a_m\}$ and $B = \{b_1, \ldots, b_n\}$, does there exist any string u so that u is both a concatenation over A and a concatenation over B?

You friend announces, "At least the problem is in NP, since I would just have to exhibit such a string u in order to prove the answer is 'yes.' " You point out — politely, of course — that this is a completely inadequate explanation; how do we know that the shortest such string u doesn't have length exponential in the size of the input, in which case it would not be a polynomial-size certificate?

However, it turns out that this claim can be turned into a proof of membership in NP. Specifically, prove the following statement:

> If there is a string u that is a concatenation over both A and B, then there is such a string whose length is bounded by a polynomial in the sum of the lengths of the strings in $A \cup B$.