Low Precision Arithmetic

CS6787 Lecture 10 — Fall 2021
Memory as a Bottleneck

• So far, we’ve just been talking about **compute**
  • e.g. techniques to decrease the amount of compute by decreasing iterations

• But machine learning systems need to process **huge amounts of data**

• Need to **store, update, and transmit** this data

• As a result: **memory** is of critical importance
  • Many applications are memory-bound
Memory: The Simplified Picture
Memory: The Multicore Picture
Memory: The Distributed Picture

Network

GPU

GPU
What can we learn from these pictures?

• Many more memory boxes than compute boxes
  • And even more as we zoom out

• Memory has a hierarchical structure

• Locality matters
  • Some memory is closer and easier to access than others
  • Also have standard concerns for CPU cache locality
What limits us?

• **Memory capacity**
  • How much data can we store locally in RAM and/or in cache?

• **Memory bandwidth**
  • How much data can we load from some source in a fixed amount of time?

• **Memory locality**
  • Roughly, how often is the data that we need stored nearby?

• **Power**
  • How much energy is required to operate all of this memory?
One way to help:
Low-Precision Arithmetic
Low-Precision Arithmetic

• Traditional ML systems use 32-bit or 64-bit floating point numbers

• But do we actually need this much precision?
  • Especially when we have inputs that come from noisy measurements

• Idea: instead use 8-bit or 16-bit numbers to compute
  • Can be either floating point or fixed point
  • On an FPGA or ASIC can use arbitrary bit-widths
Low Precision and Memory

• Major benefit of low-precision: *uses less memory bandwidth*

<table>
<thead>
<tr>
<th>Precision in DRAM</th>
<th>Memory Throughput</th>
</tr>
</thead>
<tbody>
<tr>
<td>64-bit float vector</td>
<td>5 numbers/ns</td>
</tr>
<tr>
<td>32-bit float vector</td>
<td>10 numbers/ns</td>
</tr>
<tr>
<td>16-bit int vector</td>
<td>20 numbers/ns</td>
</tr>
<tr>
<td>8-bit int vector</td>
<td>40 numbers/ns</td>
</tr>
</tbody>
</table>

(assuming ~40 GB/sec memory bandwidth)
Low Precision and Memory

- Major benefit of low-precision: takes up less space

**Precision in DRAM**
- 64-bit float vector
- 32-bit float vector
- 16-bit int vector
- 8-bit int vector

**Cache Capacity**
- 4 M numbers
- 8 M numbers
- 16 M numbers
- 32 M numbers

(assuming ~32 MB cache)
Low Precision and Parallelism

- Another benefit of low-precision: use **SIMD instructions** to get more parallelism on CPU

**SIMD Precision**
- 64-bit float vector
  - F64 F64 F64 F64
- 32-bit float vector
  - F32 F32 F32 F32 F32 F32
- 16-bit int vector
  - 16-bit int vector
- 8-bit int vector
  - 8-bit int vector

**SIMD Parallelism**
- 4 multiplies/cycle
  - (vmulpd instruction)
- 8 multiplies/cycle
  - (vmulps instruction)
- 16 multiplies/cycle
  - (vpmaddwd instruction)
- 32 multiplies/cycle
  - (vpmaddubsw instruction)
Low Precision and Power

- Low-precision computation can even have a super-linear effect on energy

- Memory energy can also have quadratic dependence on precision
Effects of Low-Precision Computation

• **Pros**
  • Fit more numbers (and therefore more training examples) in memory
  • Store more numbers (and therefore larger models) in the cache
  • Transmit more numbers per second
  • Compute faster by extracting more parallelism
  • Use less energy

• **Cons**
  • Limits the numbers we can represent
  • Introduces *quantization error* when we store a full-precision number in a low-precision representation
Numeric Formats in Machine Learning

How do we represent numbers as bit patterns on a computer?
A representative setup: DNN training

Many of the large-scale learning tasks we want to accelerate are deep learning tasks.

A **deep neural network (DNN)** looks like this:

Many layers connected to each other in series.

To train, we compute the loss gradient and run stochastic gradient descent:

$$w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t)$$
A representative setup: DNN training

- Standard method of computing gradient for SGD uses backpropagation.
- Computationally, it looks like this on the level of a single layer:

\[
\text{Weights} \downarrow \text{activations} \downarrow \text{activations}_{\text{prev}} \uparrow \text{weights} \uparrow \text{weights \_ accumulator} \downarrow \text{weights \_ gradient} \uparrow \text{backwards} \downarrow \text{storage} \uparrow \text{backwards}_{\text{next}}
\]

All of the signals here are vectors of real numbers. But how are they stored on a computer?
The standard approach
Single-precision floating point (FP32)

- 32-bit floating point numbers

| 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| sign | 8-bit exponent | 23-bit mantissa |

• Usually, the represented value is

represented number = \((-1)^{\text{sign}} \cdot 2^{\text{exponent}-127} \cdot 1.b_{22}b_{21}b_{20} \ldots b_0\)

• Has a machine epsilon (measures relative error) of \(\epsilon_{\text{machine}} \approx 6.0 \times 10^{-8}\)
An example

• Let’s convert the number 6.5 to floating point.

\[ 6.5 = 13 \times 2^{-1} = (8 + 4 + 1) \times 2^{-1} \]
\[ = 1101_b \times 2^{-1} = 1.101_b \times 2^2 \]
\[ = 1.101_b \times 2^{(129-127)} \]
\[ = 1.101_b \times 2^{(10000001!b^{-127})} \]

1 10000000 1010000000000000000000000
What is the machine epsilon?

• Represents the relative error of the floating-point format
  • One half the distance between 1 and the next-largest floating point number
  • If there are $m$ mantissa bits, $\varepsilon_{\text{machine}} \approx 2^{-m-1}$
  • Because the smallest representable number $> 1$ is $1 + 2^{-m}$

Relative error bound. If $x \in \mathbb{R}$ is any number in range of the format, and $\hat{x}$ is the nearest number representable in the format, then

$$|\hat{x} - x| \leq \varepsilon_{\text{machine}} \cdot |x|.$$  

Similarly, if $x, y \in \mathbb{R}$ are two floating-point numbers, $\star$ is any primitive numerical operation (e.g. $+$, $\times$, etc.), and $\oplus$ is the floating-point “version” of that op, then

$$|(x \oplus y) - (x \star y)| \leq \varepsilon_{\text{machine}} \cdot |x \star y|.$$  

Or, confusingly, twice this.
A low-precision alternative
**FP16/Half-precision floating point**

- 16-bit floating point numbers

![Binary Representation]

- Usually, the represented value is

\[ x = (-1)^{\text{sign bit}} \cdot 2^{\text{exponent}-15} \cdot 1.\text{significand}_2 \]
Numeric properties of 16-bit floats

- A larger machine epsilon (larger rounding errors) of \( \varepsilon_{\text{machine}} = 4.9 \times 10^{-4} \)
  - Compare 32-bit floats which had \( \varepsilon_{\text{machine}} \approx 6.0 \times 10^{-8} \)

- A smaller overflow threshold (easier to overflow) at about \( 6.5 \times 10^{4} \)
  - Compare 32-bit floats where it’s \( 3.4 \times 10^{38} \)

- A larger underflow threshold (easier to underflow) at about \( 6.0 \times 10^{-8} \).
  - Compare 32-bit floats where it’s \( 1.4 \times 10^{-45} \)

With all these drawbacks, does anyone use this?
Half-precision floating point support

- Supported on most **modern machine-learning-targeted GPUs**
  - E.g. efficient implementation as far back as NVIDIA Pascal GPUs

<table>
<thead>
<tr>
<th>GPU</th>
<th>DFMA (FP64 TFLOP/s)</th>
<th>FFMA (FP32 TFLOP/s)</th>
<th>HFMA2 (FP16 TFLOP/s)</th>
<th>DP4A (INT8 TIOP/s)</th>
<th>DP2A (INT16/8 TIOP/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GP100 (Tesla P100 NVLink)</td>
<td>5.3</td>
<td>10.6</td>
<td>21.2</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>GP102 (Tesla P40)</td>
<td>0.37</td>
<td>11.8</td>
<td>0.19</td>
<td>43.9</td>
<td>23.5</td>
</tr>
<tr>
<td>GP104 (Tesla P4)</td>
<td>0.17</td>
<td>8.9</td>
<td>0.09</td>
<td>21.8</td>
<td>10.9</td>
</tr>
</tbody>
</table>

Table 1: Pascal-based Tesla GPU peak arithmetic throughput for half-, single-, and double-precision fused multiply-add instructions, and for 8- and 16-bit vector dot product instructions. (Boost clock rates are used in calculating peak throughputs. TFLOP/s: Tera Floating-point Operations per Second. TIOP/s: Tera Integer Operations per Second. https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

- Good empirical results for **deep learning**

Another common option
Bfloat16 — “brain floating point”

- Another 16-bit floating point number
- Main benefit: numeric range is now the same as single-precision float
  - Since it looks like a truncated 32-bit float
  - This is useful because ML applications are more tolerant to quantization error than they are to overflow
An alternative to low-precision floating point
Fixed point numbers

• \( p + q + 1 \) -bit fixed point number

\[ x = (-1)^{\text{sign}} \cdot \left( \text{integer part} + 2^{-q} \cdot \text{fractional part} \right) = 2^{-q} \cdot \text{whole thing as signed integer} \]
Arithmetic on fixed point numbers

- **Simple and efficient**
  - Can just use preexisting integer processing units
  - **Lower power** than floating point operations with the same number of bits

- **Mostly exact**
  - Can always convert to a higher-precision representation to avoid overflow

- Can represent a much narrower range of numbers than float

- Has an absolute error bound, not relative error bound
Support for fixed-point arithmetic

• **Anywhere integer arithmetic is supported**
  - CPUs, GPUs
  - Although not all GPUs support 8-bit integer arithmetic
  - And AVX2 does not have all the 8-bit arithmetic instructions we’d like

• Particularly effective on **FPGAs and ASICs**
  - Where floating point units are costly

• Sadly, very **little support for other precisions**
  - **4-bit operations** would be particularly useful
A powerful hybrid approach
Block Floating Point

• Motivation: when storing a vector of numbers, often these numbers all lie in the same range.
  • So they will have the same or similar exponent, if stored as floating point.

• Block floating point shares a single exponent among multiple numbers.

8-bit shared exponent
A more specialized approach

Custom Quantization Points

• Even more generally, we can just have a list of $2^b$ numbers and say that these are the numbers a particular low-precision string represents
  • We can think of the bit string as indexing a number in a dictionary

• Gives us total freedom as to range and scaling
  • But computation can be tricky

• Some research into using this with hardware support
  • “The ZipML Framework for Training Models with End-to-End Low Precision: The Cans, the Cannots, and a Little Bit of Deep Learning” (Zhang et al 2017)
Low-precision formats in general

• These are some of the most common formats used in ML
  • ...but we’re not limited to using only these formats!

• There are many other things we could try
  • For example, floating point numbers with different exponent/mantissa sizes
  • Fixed point numbers with nonstandard widths

• Problem: there’s no hardware support for these other things yet, so it’s hard to get a sense of how they would perform.
  • Need to simulate
Other Numerical Formats Used Rarely

• **BigFloats**
  • Higher-precision floating-point numbers that are implemented in software
  • Are sometimes necessary when you need very high precision, such as for very poorly conditioned problems

• Exact arithmetic with **rational numbers**
  • Lets you do arithmetic with no error
  • Numbers have variable length, because they require arbitrarily large integers
  • Can also support countable **field extensions of the rational numbers**
  • But these are very rarely used because of performance implications
Low-Precision SGD

Using low-precision arithmetic for training
How is precision used for training

- Recall our training diagram
  - Each of these signals forms a class of numbers
  - Generally, we assign a precision to each of the classes, and different classes can have different precisions

Number classes extended from “Understanding and Optimizing Asynchronous Low-Precision Stochastic Gradient Descent,” ISCA 2017:

- Dataset numbers
- Model/weight numbers
- Gradient numbers
- Communication numbers
- Activation numbers
- Backward pass numbers
- Weight accumulator
- Linear layer accumulator
Quantize classes independently

• Using low-precision for different number classes has different effects on throughput.
  • Quantizing the dataset numbers improves memory capacity and overall training example throughput

• Quantizing the model numbers improves cache capacity and saves on compute

• Quantizing the gradient numbers saves compute

• Quantizing the communication numbers saves on expensive inter-worker memory bandwidth
Quantize classes independently

- Using low-precision for different number classes has **different effects on statistical efficiency and accuracy**.
  - Quantizing the *dataset numbers* means you’re solving a different problem

- Quantizing the *model numbers* adds noise to each gradient step, and often means you can’t exactly represent the solution

- Quantizing the *gradient numbers* can add errors to each gradient step

- Quantizing the *communication numbers* can add errors which cause workers’ local models to diverge, which slows down convergence
Theoretical Guarantees for Low Precision

• Reducing precision adds noise in the form of round-off error.

• Two approaches to rounding:
  • biased rounding – round to nearest number
  • unbiased rounding – round randomly: $E[Q(x)] = x$

Using this, we can prove guarantees that SGD converges with a low precision model.
Why unbiased rounding?

• Imagine running SGD with a low-precision model with update rule

\[ w_{t+1} = \tilde{Q}(w_t - \alpha_t \nabla f(w_t; x_t, y_t)) \]

• Here, \( Q \) is an unbiased quantization function

• In expectation, this is **just gradient descent**

\[
E[w_{t+1}|w_t] = E[\tilde{Q}(w_t - \alpha_t \nabla f(w_t; x_t, y_t))|w_t] \\
= E[w_t - \alpha_t \nabla f(w_t; x_t, y_t)|w_t] \\
= w_t - \alpha_t \nabla f(w_t)
\]
Implementing unbiased rounding

• To implement an unbiased to-integer quantizer:
  sample $u \sim \text{Unif}[0, 1]$, then set $Q(x) = \lfloor x + u \rfloor$

• Why is this unbiased?

$$
E[Q(x)] = \lfloor x \rfloor \cdot P(Q(x) = \lfloor x \rfloor) + (\lfloor x \rfloor + 1) \cdot P(Q(x) = \lfloor x \rfloor + 1)
= \lfloor x \rfloor + P(Q(x) = \lfloor x \rfloor + 1) = \lfloor x \rfloor + P(\lfloor x + u \rfloor = \lfloor x \rfloor + 1)
= \lfloor x \rfloor + P(x + u \geq \lfloor x \rfloor + 1) = \lfloor x \rfloor + P(u \geq \lfloor x \rfloor + 1 - x)
= \lfloor x \rfloor + 1 + (\lfloor x \rfloor + 1 - x) = x.
$$
Doing unbiased rounding efficiently

• We still need an efficient way to do unbiased rounding

• **Pseudorandom number generation can be expensive**
  • E.G. doing C++ rand or using Mersenne twister takes many clock cycles

• Empirically, we can use **very cheap** pseudorandom number generators
  • And still get good statistical results
  • For example, we can use XORSHIFT which is just a cyclic permutation
Benefits of Low-Precision Computation

Drawbacks of low-precision

- The drawback of low-precision arithmetic is the **low precision**!

- Low-precision computation means we accumulate **more rounding error** in our computations

- These rounding errors can add up throughout the learning process, resulting in **less accurate learned systems**

- The trade-off of low-precision: **throughput/memory vs. accuracy**
Using low precision improves backpropagation as with logistic regression, is the kernel support vector machine (SVM). We hypothesized that, training at 8-bit precision is too inaccurate [9, 14]. train accurately even below 8-bits, using unbiased rounding. This is the case, and we show in Figure 7b that it is possible to achieve results with similar accuracy. Our experiments show that this is in fact the case, and that our optimizations are effective in this setting. We evaluated our techniques with various datasets, including MNIST and CIFAR10. For both these datasets, we observed speedups similar to those in Figure 7a: for both MNIST and CIFAR10, we observed performance improvements with lower precision arithmetic of arbitrary bit widths. Since this simulation test error for LeNet, a successful CNN architecture [15], has higher throughput (GNPS) while producing results with similar accuracy. To see how this affects of running on an FPGA, and validation of approach via alternate applications to convolutional neural network (CNN) base gcc full-precision baseline (gcc), we ran ten such SVM classification tasks. Convolution layers for these datasets have speedups similar to those in Figure 7a: for both MNIST and CIFAR10 [16], a deep learning library, to simulate low-precision training at 8-bit precision is too inaccurate [9, 14]. train accurately even below 8-bits, using unbiased rounding. This is the case, and we show in Figure 7b that it is possible to achieve results with similar accuracy. Our experiments show that this is in fact the case, and that our optimizations are effective in this setting. We evaluated our techniques with various datasets, including MNIST and CIFAR10. For both these datasets, we observed performance improvements with lower precision arithmetic of arbitrary bit widths.

(b) Test accuracies of low-precision SGD on LeNet neural network after 5000 passes, for various datasets.
Demo
Memory Locality and Scan Order
Memory Locality: Two Kinds

• Memory locality is needed for *good cache performance*

• **Temporal locality**
  • Frequency of reuse of the same data within a short time window

• **Spatial locality**
  • Frequency of use of data nearby data that has recently been used

• *Where is there locality in stochastic gradient descent?*
Problem: no dataset locality across iterations

• The training example at each iteration is chosen randomly
  • Called a random scan order
  • Impossible for the cache to predict what data will be needed

\[ w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t, y_t) \]

• Idea: process examples in the order in which they are stored in memory
  • Called a systematic scan order or sequential scan order
  • Does this improve the memory locality?
Random scan order vs. sequential scan order

• **Much easier to prove theoretical results** for random scan

• But **sequential scan has better systems performance**

• In practice, **almost everyone uses sequential scan**
  • There’s no empirical evidence that it’s statistically worse in most cases
  • Even though we can construct cases where using sequential scan does harm the convergence rate
Other scan orders

• **Shuffle-once**, then sequential scan
  • Shuffle the data once, then systematically scan for the rest of execution
  • Statistically very similar to random scan at the state

• **Random reshuffling**
  • Randomly shuffle on every pass through the data
  • Gets better upper bounds for SGD
    • e.g. for convex, gets $O(1/t)$ versus $O(1/\sqrt{t})$
  • Very commonly used with deep learning
Demo
Questions?

• Upcoming things
  • Project feedback coming soon!