(One) Definition of Learning

• Definition [Mitchell]:

  A computer program is said to learn from
  • experience E with respect to some class of
  • tasks T and
  • performance measure P,
  if its performance at tasks in T, as measured by P, improves with experience E.
Supervised (Batch) Learning

- **Task:**
  - Learn (to imitate) a function \( f: X \rightarrow Y \) (i.e. given \( x \), predict \( y \))

- **Experience:**
  - Learning algorithm is given the correct value of the function for particular inputs \( \rightarrow \) training examples (see table above)
  - An example is a pair \( (x, y) \), where \( x \) is the input and \( y=f(x) \) is the output of the function applied to \( x \).

- **Performance Measure:**
  - Find a function \( h: X \rightarrow Y \) predicts the same \( y \) as \( f: X \rightarrow Y \) as often as possible.

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Hypothesis Space

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Instance Space \( X \): Set of all possible objects described by attributes.

Target Function \( f \) (hidden): Maps each instance \( x \in X \) to target label \( y \in Y \).

Hypothesis \( h \): Function that approximates \( f \).

Hypothesis Space \( H \): Set of functions we consider for approximating \( f \).

Training Data \( S \): Sample of instances labeled with target function \( f \).
A Simple Strategy for Learning

• Strategy (later to be refined and justified):
  Remove any hypothesis from consideration that is not consistent with the training data.

• Can compute:
  – A hypothesis $h \in H$ such that $h(x) = f(x)$ for all $x \in S$.

• Ultimate Goal:
  – A hypothesis $h \in H$ such that $h(x) = f(x)$ for all $x \in X$. 
**Definition:** A hypothesis $h$ is **consistent** with a set of training examples $S$ if and only if $h(x) = y$ for each training example $(x, y) \in S$.

$$\text{Consistent}(h, S) \equiv [\forall (x, y) \in S : h(x) = y]$$
Definition: The version space, $V_{H,S}$, with respect to hypothesis space $H$ and training examples $S$, is the subset of hypotheses from $H$ consistent with all training examples in $S$.

$$V_{H,S} = \{ h \in H | \text{Consistent}(h, S) \}$$
List-Then-Eliminate Algorithm

• Init VS ← H
• For each training example (x, y) ∈ S
  – remove from VS any hypothesis h for which h(x) ≠ y
• Output VS
Hypothesis Space of Decision Trees

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Top-Down Induction of DT
(simplified)

Training Data: \( S = ((\tilde{x}_1, y_1), \ldots, (\tilde{x}_n, y_n)) \)

\[
\text{TDIDT}(S, y_{\text{def}})
\]

- IF(all examples in \( S \) have same class \( y \))
  - Return leaf with class \( y \) (or class \( y_{\text{def}} \) if \( S \) is empty)
- ELSE
  - Pick \( A \) as the “best” decision attribute for next node
  - FOR each value \( v_i \) of \( A \) create a new descendent of node
    - \( S_i = \{ (\tilde{x}, y) \in S : \text{attribute } A \text{ of } \tilde{x} \text{ has value } v_i \} \)
    - Subtree \( t_i \) for \( v_i \) is \( \text{TDIDT}(S_i, y_{\text{def}}) \)
  - RETURN tree with \( A \) as root and \( t_i \) as subtrees
TDIDT($S,y_{\text{def}}$)

- **IF** (all ex in $S$ have same $y$)
  - Return leaf with class $y$
    (or class $y_{\text{def}}$, if $S$ is empty)
- **ELSE**
  - Pick $A$ as the “best” decision attribute for next node
  - FOR each value $v_i$ of $A$ create a new descendent of node
    - $S_i = \{(\vec{x}, y) \in S : \text{attr } A \text{ of } \vec{x} \text{ has val } v_i}\}$
    - Subtree $t_i$ for $v_i$ is TDIDT($S_i,y_{\text{def}}$)
  - RETURN tree with $A$ as root and $t_i$ as subtrees
Which Attribute is “Best”? 

A1

true → [21+, 5-]
false → [8+, 30-]

A2

true → [18+, 33-]
false → [11+, 2-]
Example: Text Classification

• Task: Learn rule that classifies Reuters Business News
  – Class +: “Corporate Acquisitions”
  – Class -: Other articles
  – 2000 training instances

• Representation:
  – Boolean attributes, indicating presence of a keyword in article
  – 9947 such keywords (more accurately, word “stems”)

LAROCHE STARTS BID FOR NECO SHARES
Investor David F. La Roche of North Kingstown, R.I., said he is offering to purchase 170,000 common shares of NECO Enterprises Inc at 26 dlr each. He said the successful completion of the offer, plus shares he already owns, would give him 50.5 pct of NECO's 962,016 common shares. La Roche said he may buy more, and possible all NECO shares. He said the offer and withdrawal rights will expire at 1630 EST/2130 gmt, March 30, 1987.

SALANT CORP 1ST QTR
FEB 28 NET
Oper shr profit seven cts vs loss 12 cts.
Oper net profit 216,000 vs loss 401,000. Sales 21.4 mln vs 24.9 mln.
NOTE: Current year net excludes 142,000 dlr tax credit. Company operating in Chapter 11 bankruptcy.
Decision Tree for “Corporate Acq.”

Learned tree:
- has 437 nodes
- is consistent

Accuracy of learned tree:
- 11% error rate on test sample

Note: word stems expanded for improved readability.
Overfitting

- Note: Accuracy = 1.0 - Error

[Mitchell]