Prove that $\mathcal{R}$ is a system in which the agents have perfect recall iff the following condition holds: for all points $(r, m)$ and $(r', m')$ and all agents $i$, if $(r, m) \sim_i (r', m')$ and $k \leq m$, then there exists $k' \leq m'$ such that $(r, k) \sim_i (r', k')$. (Hint: to prove that this condition implies perfect recall, use induction on the sum of the lengths of the local-state sequence at $(r, m)$ and $(r', m')$. Specifically, show by induction on $N$ that if the condition holds, $(r, m) \sim_i (r', m')$, and sum of the length of $i$’s local-state sequence at $(r, m)$ and the length of $i$’s local-state sequence at $(r', m')$ is at most $N$, then $i$ has the same local-state sequence at $(r, m)$ and $(r', m')$.)