Prove that $\mathcal{R}$ is a system in which the agents have perfect recall iff the following condition holds: for all points (r, m) and (r', m') and all agents i, if $(r, m) \sim_i (r', m')$ and $k \leq m$, then there exists $k' \leq m'$ such that $(r, k) \sim_i (r', k')$. (Hint: to prove that this condition implies perfect recall, use induction on the sum of the lengths of the local-state sequence at (r, m) and (r', m'). Specifically, show by induction on N that if the condition holds, $(r, m) \sim_i (r', m')$, and sum of the length of i's local-state sequence at (r, m) and the length of i's local-state sequence at (r', m') is at most N, then i has the same local-state sequence at (r, m) and (r', m').)