A Knowledge-Based Analysis of the Blockchain

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The blockchain

At the heart of bitcoin is a *blockchain*, protocol for achieving consensus on a public ledger that records bitcoin transactions.

- Blockchain protocols can be used for applications like contract signing and for making transactions (like house sales) public.
- Contract signing is supposed to give agent *common knowledge*
  - Both signers know that both signers know . . . that the contract was signed
- Similarly, make a house sale public means make the sale common knowledge.

What is the semantics of a blockchain protocol?

- What properties do we want it to guarantee?
- **Claim:** these questions are best understood in terms of knowledge
Why it’s subtle

A ledger is a distributed database that can be viewed as a sequence of blocks of data.

- Different agents typically have different views about which transactions are in the blockchain.
- With current blockchain protocols, it is also possible that a given transaction is included in agent $i$’s view of the ledger at time $m$ and not included at a later time $m'$.
- The set of agents involved changes over time.
- We need to allow for dishonest agents that do not follow the protocol, and may try to subvert it.
- We have asynchrony:
  - message delivery time is uncertain (although bounded)

We need to guarantee that a blockchain protocol gives us appropriate knowledge despite all this.
A ledger $X$ is a $T$-\textit{prefix} of a ledger $Y$ if $X$ is any prefix of the ledger that contains all but the last $T$ transactions in $Y$. 

Blockchain protocols are assumed to be $T$-\textit{consistent}: if $i$ is honest (i.e., $i$ has followed the protocol since joining the system) and $X$ is a $T$-prefix of $i$'s ledger at time $m$, then at all times $m' \geq m$, all honest agents will have $X$ as a prefix of their ledger.

Does $T$-consistency suffice to use a blockchain protocol for the types of applications envisioned for it? 

\textit{Spoiler alert: no!}

So what else do we need? 

That depends on what we want to achieve.
Typical assumptions

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A contract-signing example

- Suppose that attorneys require that electronic signatures on the contract are received by 11:30 AM on a global clock
- If they are received by then, the contract will be in force at noon on the global clock.

We might hope that if signatures are received by 11:30 AM, it is common knowledge that messages from the attorney are all received within at most 5 minutes, and everything is recorded on the ledger, then at noon on the global clock all agents will have common knowledge that the contract is in force.
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Unfortunately, this does not follow from $T$-consistency:
- If $T = 10$ and the only transactions are the receipt of the messages and the contract being signed, it is compatible with $T$-consistency that the contract being signed is on one agent’s ledger but never gets on the second agent’s ledger.
\(\Delta\)-weak growth

We need one more property to deal with this example:

- \(\Delta\)-weak growth [Pass-Seeman-Shelat 2016]: if \(i\) is an honest agent and has a ledger of length \(N\) at time \(t\), then all honest agents will have ledgers of length \(N\) by time \(t + \Delta\).
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- \(\Delta\)-weak growth [Pass-Seeman-Shelat 2016]: if \(i\) is an honest agent and has a ledger of length \(N\) at time \(t\), then all honest agents will have ledgers of length \(N\) by time \(t + \Delta\).

Our main result: the combination of \(\Delta\)-weak growth and \(T\)-consistency suffices not just for agent 1 to know that agent 2 will know (within time \(\Delta\)) that 1 will have the contract in his ledger; the combination is necessary and sufficient to achieve \(\Delta\)-\(\Box\)-common knowledge among the honest agents that the contract is in all of their ledgers.

- Roughly speaking, each honest agent knows that within \(\Delta\) all the honest agents will know from that point on that within \(\Delta\) all the honest agents will know from that point on \(\ldots \phi\).
- Even though the set of honest agents can change over time

This level of knowledge suffices to ensure coordination among honest agents within a window of \(\Delta\).
Assumptions:

- A system $\mathcal{R}$ consists of a set of runs or histories
- $\mathcal{A}$ = all agents that could ever be in the system
- $\mathcal{A}(r, m)$ = the agents actually present in history $r$ at time $m$.
- $\mathcal{H}(r, m) \subseteq \mathcal{A}(r, m)$ consists of the honest agents at $(r, m)$
  - $\mathcal{H}$ and $\mathcal{A}$ are indexical sets;
  - they can shrink or grow over time
- At $(r, m)$, each agent in $\mathcal{A}(r, m)$ is in some local state
- The global state at $(r, m)$ is $\{(s_i, i) : i \in \mathcal{A}(r, m)\}$
  - The set of local states of agents $i \in \mathcal{A}(r, m)$
- Let $r_i(m) = s_i$ (for $i \in \mathcal{A}(r, m)$)
Interpreted systems

To reason about a blockchain protocol, we start with primitive propositions

- \( i \in \mathcal{H} : (I, r, m) \models i \in \mathcal{H} \) if \( i \in \mathcal{H}(r, m) \)

- \( T\text{-}\text{prefix}(X, L_i) : (I, r, m) \models T\text{-}\text{prefix}(X, L_i) \) if \( X \) is a \( T\text{-}\text{prefix} \) of \( L_i(r, m) \), \( i \)'s view of the ledger at time \( m \) in run \( r \)
Interpreted systems

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- $i \in \mathcal{H}$: $(I, r, m) \models i \in \mathcal{H}$ if $i \in \mathcal{H}(r, m)$
- $T$-prefix($X, L_i$): $(I, r, m) \models T$-prefix($X, L_i$) if $X$ is a $T$-prefix of $L_i(r, m)$, $i$’s view of the ledger at time $m$ in run $r$

Non-epistemic operators:

- $(I, r, m) \models \Box \phi$ iff $(I, r, m') \models \phi$ for all $m' \geq m$
- $(I, r, m) \models \Diamond^\Delta \phi$ iff $(I, r, m + \Delta) \models \phi$. 

Proposition: Protocol $P$ is $T$-consistent and satisfies $\Delta$-weak growth iff for all $i, j \in AG$,
the formula $i \in H \land T$-prefix($X, L_i$) $\Rightarrow \Diamond^\Delta j \in H \land T$-prefix($X, L_j$) is valid in $I_P$.

$I_P$ is the system corresponding to protocol $P$.
Interpreted systems

To reason about a blockchain protocol, we start with primitive propositions

- \( i \in \mathcal{H}: (\mathcal{I}, r, m) \models i \in \mathcal{H} \) if \( i \in \mathcal{H}(r, m) \)

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Non-epistemic operators:

- \( (\mathcal{I}, r, m) \models \lozenge \phi \) iff \( (\mathcal{I}, r, m') \models \phi \) for all \( m' \geq m \)

- \( (\mathcal{I}, r, m) \models \circlearrowright^\Delta \phi \) iff \( (\mathcal{I}, r, m + \Delta) \models \phi \).

**Proposition:** Protocol \( P \) is \( T \)-consistent and satisfies \( \Delta \)-weak growth iff for all \( i, j \in AG \), the formula

\[
i \in \mathcal{H} \land T\text{-prefix}(X, L_i) \Rightarrow \circlearrowright^\Delta \lozenge (j \in \mathcal{H} \Rightarrow T\text{-prefix}(X, L_j))
\]

is valid in \( \mathcal{I}_P \).

- \( \mathcal{I}_P \) is the system corresponding to protocol \( P \)
Epistemic operators

But what do agents *know* if they run a blockchain protocol?

Suppose that $S$ is an indexical set:

- $(\mathcal{I}, r, m) \models B_i^S \phi$ iff $(\mathcal{I}, r', m') \models \phi$ for all $(r', m')$ such that $r_i(m) = r'_i(m)$ and $i \in S(r', m')$.
- $i$ knows that if $i \in S$, then $\phi$ holds
- idea for definition due to Moses and Tuttle [1988]

- $E_S \phi = \text{def} \land_{i \in S} B_i^S \phi$
- $C_S \phi = \text{def} \land_{n=1}^{\infty} E_S^n \phi$

More general notion:

- $C_S^{\Delta \Box} \phi = \text{def} \land_{n=1}^{\infty} (\bigcirc^{\Delta \Box} E_S \phi)^n$
- $\Delta-\Box$ common knowledge among the players in $S$. 
Towards an epistemic characterization

We want to prove that, for all \( i, j \)

\[
i \in \mathcal{H} \land T\text{-prefix}(X, L_i) \Rightarrow C_{\mathcal{H}}^{\Delta} (j \in \mathcal{H} \Rightarrow T\text{-prefix}(X, L_j)).
\]

- if \( i \) is honest then everything in \( i \)'s \( T\)-prefix is \( \Delta \)-\( \square \) common knowledge among the honest players
  - within \( \Delta \), all the honest players will know that from then on, within \( \Delta \), all the honest players will know . . . everything in \( i \)'s \( T\)-prefix
Towards an epistemic characterization

We want to prove that, for all $i, j$

$$i \in \mathcal{H} \land T\text{-prefix}(X, L_i) \Rightarrow C_{\mathcal{H}}^{\bigcirc \Delta \Box} (j \in \mathcal{H} \Rightarrow T\text{-prefix}(X, L_j)).$$

- if $i$ is honest then everything in $i$’s $T$-prefix is $\Delta - \Box$ common knowledge among the honest players
  - within $\Delta$, all the honest players will know that from then on, within $\Delta$, all the honest players will know ... everything in $i$’s $T$-prefix

Standard way to prove common knowledge:

**Lemma:** $i \in \mathcal{H} \land \psi \Rightarrow \bigcirc^{\Delta} \Box E_{\mathcal{H}} \psi$ is valid for all $i \in \mathcal{H}$, then so is $i \in \mathcal{H} \land \psi \Rightarrow C_{\mathcal{H}}^{\bigcirc \Delta \Box} \psi$.

**Problem:** What is $\psi$? $T$-prefix$(X, L_i)$? $T$-prefix$(X, L_j)$

- The formulas $T$-prefix$(X, L_j)$ are different for each $j$
- But they’re similar!
  - They say “$X$ is in ‘my’ $T$-prefix”
- If we change the language slightly, they become the same!
We allow *agent-relative* formulas

- Their truth depends on the agent

Have two new primitive propositions:

- $I \in \mathcal{H}$ ("I am honest")
  - $(I, r, m, i) \models I \in \mathcal{H}$ if $i \in \mathcal{H}(r, m)$

- $T$-prefix($X, L$) ("$X$ is in a $T$-prefix of my ledger")
  - $(I, r, m, i) \models T$-prefix($X, L$) if $X$ is a $T$-prefix of $L_i(r, m)$

Can prove the validity of

$$I \in \mathcal{H} \land T$-prefix($X, L$) \Rightarrow C^{\bigcirc \Delta \Box}(T$-prefix($X, L$)).$$

This gives us the desired epistemic characterization of the blockchain protocol.
Adding probability

In practice, $T$-consistency and $\Delta$-weak growth are not guaranteed to hold.

- They are only guaranteed to hold with high probability
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We can characterize the knowledge of agents using a blockchain protocol with probabilistic beliefs by considering probabilistic variants of common knowledge

- With high probability, within $\Delta$ everybody knows from then on that with high probability, within $\Delta$ . . .

There are some subtleties in defining this in an asynchronous setting.

- See the full paper
Discussion

We got what we thought we wanted. Did we get what we needed?

- We may also want $\Delta'$-liveness.
- If $i$ wants to add something to a ledger, then within $\Delta'$ it is added.
- May want to prevent ledgers from growing too quickly.
- So that the $N$th transaction for $i$ is close to the $N$th transaction for $j$.

But for many contract signing applications, $\Delta'$-common knowledge is just what we need.
Discussion

We got what we thought we wanted. Did we get what we needed?

Not necessarily:

▶ We may also want \( \Delta'\)-liveness
  ▶ If \( i \) wants to add something to a ledger, then within \( \Delta' \) it is added

▶ May want to prevent ledgers from growing too quickly
  ▶ So that the \( N \)th transaction for \( i \) is close to the \( N \)th transaction for \( j \)

But for many contract signing applications, \( \Delta\)-\( \square \) common knowledge is just what we need.
Example: Suppose that two players want to sign a contract if either gets some signal (in their ledger).

- If both sign within some small interval $\Delta$ after at least one gets a signal, then they both get high utility.
- If one signs but the other doesn’t sign soon enough, both get large negative utility.
- If one player signs before a signal is received or signs without the other player signing, then that player gets large negative utility.
- A player who doesn’t sign gets utility 0.
- The signing is external to the ledger.

A player who gets a signal signs, and sends a message to the other player to sign, who signs when he gets the message.

- They are signing when $\Delta - \Box$ common knowledge holds.