CS674, Spring 2002
Lecture Handout, 4/1/02

1. From Claude E. Shannon, "A Mathematical Theory of Communication", *The Bell System Technical Journal*, July 1948:

> The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages.

Note Weaver's comment in the his prefatory paper (see *The Mathematical Theory of Communication*, Shannon and Weaver, 1949): "But this does not mean that the engineeering aspects are necessarily irrelevant to the semantic aspects".

2. Some quantities.

2**(a)** Entropy. Let X be a discrete random variable with alphabet $\{x_1, \ldots, x_n\}$ and probability mass function $p(x)$. $H(X)$ (or $H(p)$) is defined as

$$H(p) = -\sum_{i=1}^{n} p(x_i) \log p(x_i)$$

2**(b)** Cross-entropy. Let $p(x)$ and $q(x)$ be two pmfs.

$$H(p,q) = -\sum_{i=1}^{n} p(x_i) \log q(x_i)$$

2**(c)** Relative entropy or Kullback-Leibler (KL) divergence.

$$D(p||q) = \sum_{i=1}^{n} p(x_i) \log \frac{p(x_i)}{q(x_i)}$$