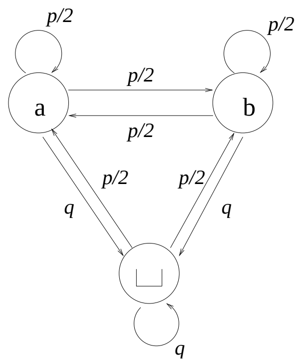
CS674, Spring 2002
Lecture Handout, 2/11/02

1. Markov chain parameters and notation. We use t to denote the time step, $t \in \{1, 2, 3, \ldots\}$.

$\mathcal{S} = \{s_1, \ldots, s_n\}$: finite state set, with cardinality $n > 0$.
Q_t: random variable for state at time t.
$P(s_i)$: Initial probability $P(Q_1 = s_i), s_i \in \mathcal{S}$
$P(s_j|s_i)$: transition probability $P(Q_{t+1} = s_j|Q_t = s_i)$, $s_i, s_j \in \mathcal{S}$.

2. Markov chain for Miller's monkey with a two-letter keyboard, assuming the monkey types forever rather than stopping after the first word (as we implicitly assumed last time). We have $q = 1 - p$, $P(\text{a}) = P(\text{b}) = p/2$ and $P(\text{“space”}) = q$). (We have to define “generating a word” more carefully, to deal with starting spaces and the like. We might also like to prevent multiple spaces in a row.)



3. Hidden Markov model parameters: Markov chain parameters plus the following:

$\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$: finite output alphabet, $m > 0$.
We enforce $\Sigma \cap \mathcal{S} = \emptyset$.
O_t: random variable for output at time t
$P(\sigma_j|s_i)$: output probability $P(O_t = \sigma_j|Q_t = s_i)$, $\sigma_j \in \Sigma$, $s_i \in \mathcal{S}$

4. Forward probability for a given output $o = o_1 o_2 \cdots o_\ell$ and a given HMM (with parameters θ, but we'll suppress θ for clarity):

$$\begin{aligned} \alpha_t(s_i) &= P(o_1 o_2 \cdots o_t, s_i) \\ &= P(O_1 = o_1, O_2 = o_2 \cdots O_t = o_t, Q_t = s_i) \end{aligned}$$

5. Underlying Markov chain for an HMM modeling Miller's monkey. We have $q = 1 - p$, $P(\text{letter}) = 1$ (or, alternatively, $P(\text{letter}) = p$). (As before, we might also like to prevent multiple spaces in a row.)