CS6702
Computational Sustainability
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by

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Objectives of This Lecture

• Present the basic bioeconomic model from Colin Clark's *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, Wiley-Interscience, New York, 1990.

• Present the stochastic model from Bill Reed's "Optimal Escapement Levels in Stochastic and Deterministic Harvesting Models," *Journal of Environmental Economics and Management*, 6:350-363, 1979.

• Suggest some open questions and models that might serve as potential projects for CS6702.

The Basic Bioeconomic Model from Clark (1990)

$X = X(t)$ the biomass of a renewable resource at instant t,

$Y = Y(t)$ harvest of the renewable resource at instant t,

$F(X)$ a net growth function,

$dX/dt = \dot{X} = F(X) - Y$ resource dynamics (state equation),

$\pi = \pi(t) = \pi(X, Y)$ net benefit of harvest Y from X,

$e^{-\delta t}$ a discount factor, $\delta > 0$ the rate of discount.

Is exponential discounting the result of evolution? Did evolution select hunter-gatherers who maximized the discounted sum of expected utility? See Arthur J. Robson and Larry Samuelson's "The Evolution of Time Preference with Aggregate Uncertainty," *American Economic Review*, 99(5):1925-1953, 2009.

The basic bioeconomic problem might be stated as

$$\underset{\{Y\}}{Maximize} \;\; \pi = \int_0^\infty \pi(X,Y)e^{-\delta t}dt$$

$$Subject\ to \;\; \dot{X} = F(X) - Y,\; X(0) = X_0 > 0 \; given.$$

This problem can be solved using the *Maximum Principle* where one maximizes the current-value Hamiltonian

$$H = H(t) = \pi(X,Y) + \mu[F(X) - Y]$$

at each instant in time and where $\mu = \mu(t)$ is the current-value "shadow price" for $X(t)$. The shadow price in the marginal value of $X(t)$ *in situ*. The maximized current-value Hamiltonian can be thought of as "properly accounted income." In a resource or environmental management problem it would be "green properly accounted income." See Martin Weitzman's *Income, Wealth, and the Maximum Principle*, Harvard University Press, Cambridge, 2003.

The first-order necessary conditions to the basic bioeconomic problem require

$$\partial\pi(\bullet)/\partial Y = \mu$$

$$\dot{\mu} - \delta\mu = -\partial H/\partial X = -[\partial\pi(\bullet)/\partial X + \mu F'(X)] \quad \text{(co-state equation)}$$

$$\dot{X} = \partial H/\partial\mu = F(X) - Y$$

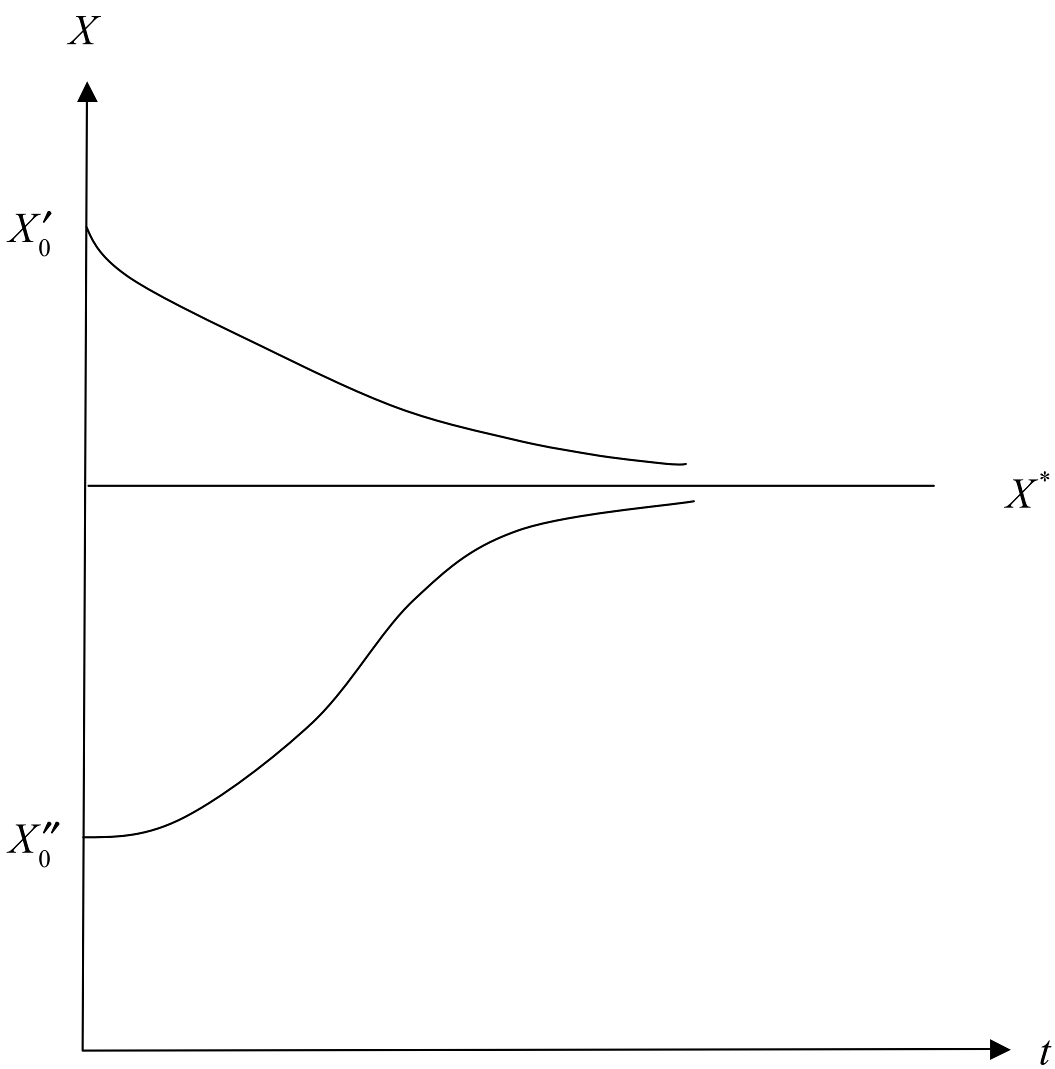
At an interior steady-state, bioeconomic optimum $[X^*, Y^*]$ will be defined by

$$F'(X) + \frac{\partial\pi(\bullet)/\partial X}{\partial\pi(\bullet)/\partial Y} = \delta$$

$$Y = F(X)$$

When the current-value Hamiltonian is strictly concave in X and Y the approach to $[X^*, Y^*]$ will be asymptotic. See the figure on the next page.

Optimal Asymptotic Approach to X*



There is an interesting special case when $F(X) = rX(1 - X/K)$, and $\pi(X,Y) = pY - cY/(qX) = [p - c/(qX)]Y$. The current-value Hamiltonian is $H = [p - c/(qX) - \mu]Y + \mu F(X)$ and is linear in Y. The sign of $\sigma(t) = p - c/(qX) - \mu$ will play a critical role in the approach to X^*. There is an analytic solution for X^* given by

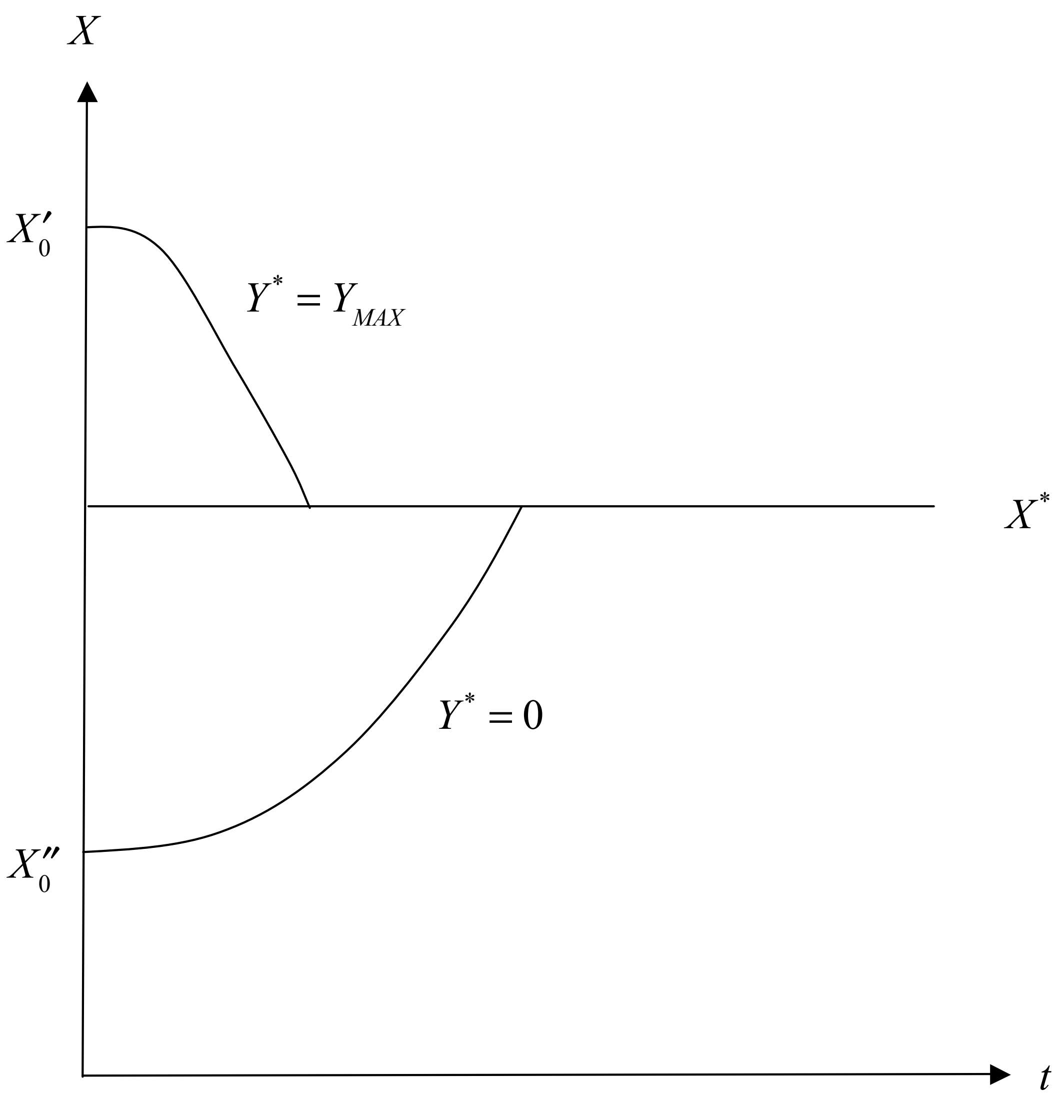
$$X^* = \left[\frac{K}{4}\right]\left[\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{Kpqr}}\right]$$

where $p > 0$ is the per unit price for Y, $c > 0$ is the unit cost of fishing effort, $q > 0$ is the "catchability coefficient," $\delta > 0$ is the rate of discount, $r > 0$ is the intrinsic growth rate, and $K > 0$ is the environmental carrying capacity. With $Y_{MAX} \geq Y \geq 0$, the approach to X^* is most rapid and

$$Y^* = \left\{\begin{matrix} Y_{MAX} \ \ if \ \ X > X^* \\ rX^*(1 - X^*/K) \ if \ \ X = X^* \\ 0 \ if \ \ X < X^* \end{matrix}\right\}$$

See the figure on the next page.

The Most Rapid Approach to X*



Bill Reed's discrete-time stochastic model.

$t = 0,1,2,...,T$ time is partitioned into discrete intervals,

X_t resource biomass in period t,

Y_t harvest in period t,

$S_t = X_t - Y_t$ *escapement* in period t,

$X_{t+1} = z_{t+1}G(S_t)$ a stochastic map with

z_{t+1} an i.i.d. random variable with $\mathrm{E}\{z\} = 1$,

$\mathrm{E}\{\bullet\}$ the expectation operator,

$\pi_t = N(X_t) - N(S_t)$ net benefit in period t, where

$N(m) = pm - (c/q)\ln(m)$ with p, c, q, as before,

$\rho = 1/(1+\delta)$ the discrete-time discount factor,

$\delta > 0$ the per period discount rate.

The optimization problem seeks to

$$\underset{\{S_t\}}{Maximize} \quad \pi = \mathrm{E}\left\{\sum_{t=0}^{T} \rho^t [N(X_t) - N(S_t)]\right\}$$

$$Subject\ to \quad X_{t+1} = z_{t+1} G(S_t),\ X_0 > 0\ given,$$

$$z_{t+1}\ an\ i.i.d.\ random\ \mathrm{var}iable\ with\ \mathrm{E}\{z\} = 1.$$

Reed uses *dynamic programming* to show that as $T \to \infty$ the optimal policy is a constant escapement policy, denoted S^*, where

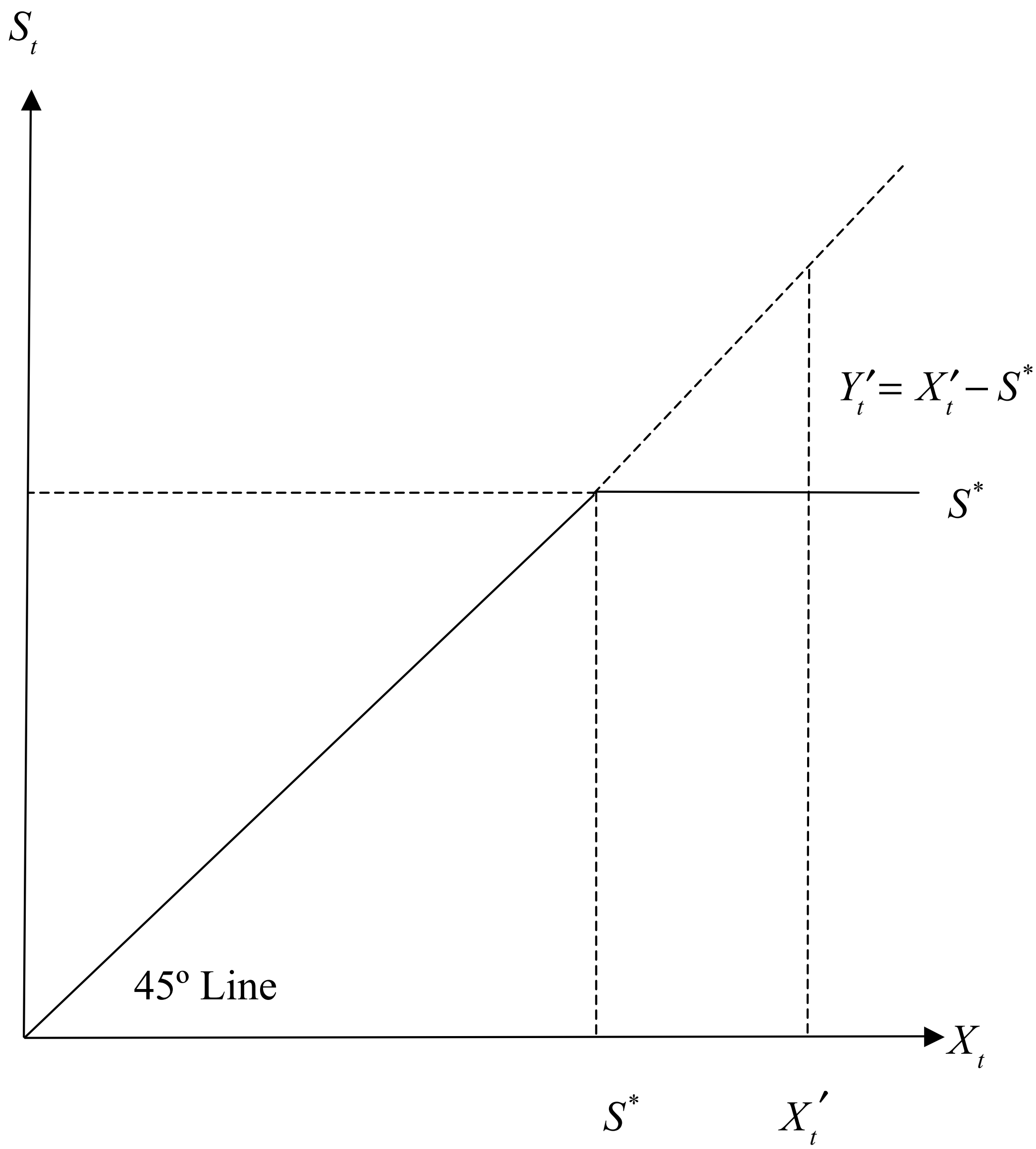
$$Y_t^* = \left\{\begin{matrix} (X_t - S^*)\ if\ X_t > S^* \\ 0\ if\ X_t \leq S^* \end{matrix}\right\}$$

and where S^* must satisfy the implicit equation

$$G'(S)\left[\frac{\mathrm{E}_z\{zN'(zG(S))\}}{N'(S)}\right] = (1+\delta)$$

See the figure on the next page.

The Optimal Constant Escapement Policy, S^*



In the Reed stochastic escapement model, there is no steady state. Resource biomass fluctuates according to an optimal stationary distribution defined by $X_{t+1} = z_{t+1}G(S_t)$ when

$$S_t = \left\{ \begin{matrix} X_t \ \ if \ \ X_t \leq S^* \\ S^* \ \ if \ \ X_t > S^* \end{matrix} \right\}$$

This is a good model for salmon in the Pacific Northwest.

Some Open Questions

1. In discrete-time models the qualitative behavior of the optimal solution can change from asymptotic convergence to periodic harvest or “pulse fishing.” Why? Can we identify a bifurcation diagram for a particular discrete-time, bioeconomic model and show how the behavior of the optimal solution changes as the values of key parameters change?

2. There are age-structured bioeconomic models. See Oli Tahvonen, "Economics of Harvesting Age-Structured Fish Populations," *Journal of Environmental Economics and Management*, 58:281-299, 2009. As Steve Elner suggested in our first class, *fish evolve*! If we harvest to many big fish the population selects for smaller fish which might not be as commercially valuable as large fish. Thus, what is the optimal size distribution and how many fish of different sizes can we harvest while maintaining the optimal size distribution as the stationary, evolutionary response by the fish population? A great species to study would be the blue fin tuna in the Mediterranean Sea or the Northwest Atlantic. Point Judith, Rhode Island, used to call itself "The Giant Blue Fin Tuna Capital of the World." Boats using spotter planes would go out to harpoon 3,000 pound, 11-foot blue fins. (That's a lot of sushi!) But no longer. As we over-harvested the giant blue fin tuna, did they select to a smaller size distribution? Evolutionary game theory anyone?

3. Dean P. Foster and Sergiu Hart, "An Operational Measure of Riskiness," *Journal of Political Economy*, 117(5):785-814 (2009), propose a measure of riskiness that is "objective." It depends only on the gamble, not on the decision maker. The measure is based on identifying, for any gamble, the critical wealth level below which it is too "risky" to accept the gamble. If wealth is below the critical level, and the decision maker continuously accepts the gamble, (s)he will go bankrupt with probability one. Can such a measure of riskiness be used to identify critical biomass levels when harvesting a renewable resource? How does the Foster and Hart measure of riskiness relate to "the precautionary principle" in environmental and resource economics?