Computational Complexity Hierarchy

EXP-complete:
games like Go, …

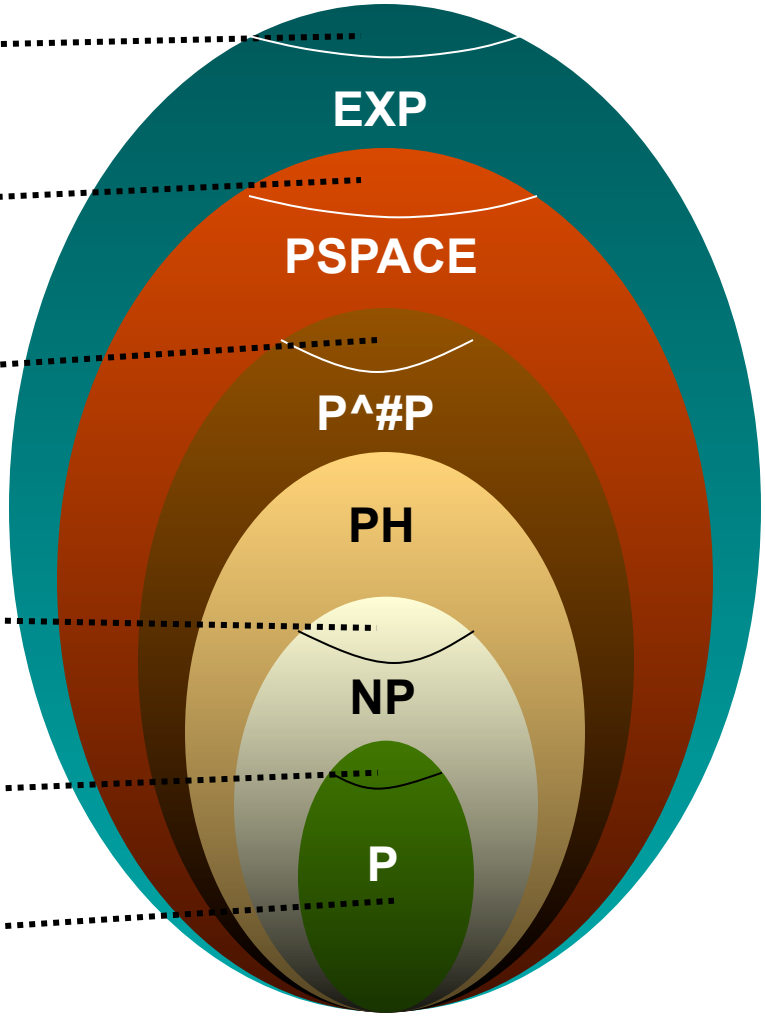
PSPACE-complete:
QBF, ***planning***, chess (bounded), …

#P-complete/hard:
#SAT, sampling, ***probabilistic inference***, …

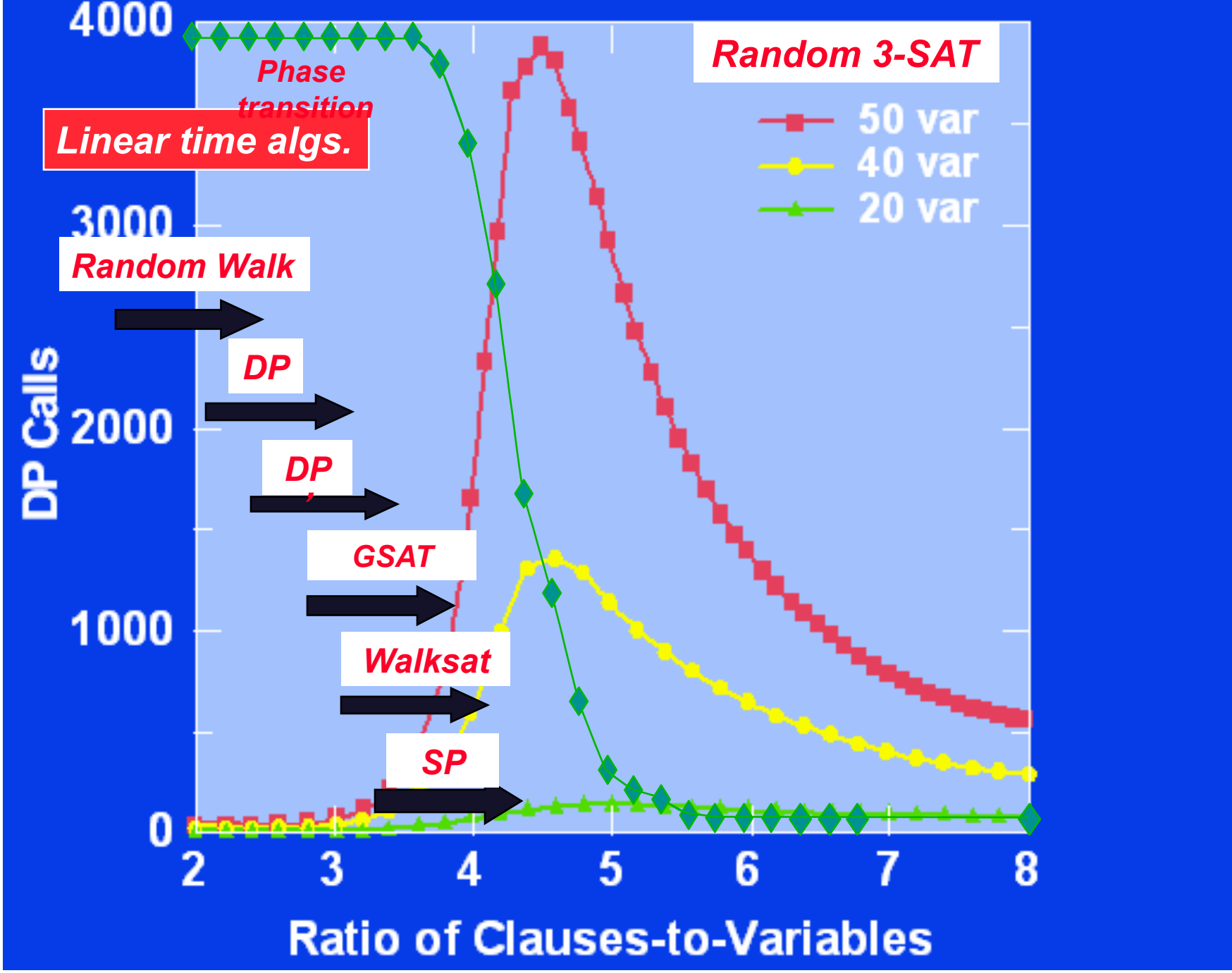
NP-complete:
SAT, ***propositional reasoning***, scheduling, graph coloring, puzzles, …

P-complete:
circuit-value, …

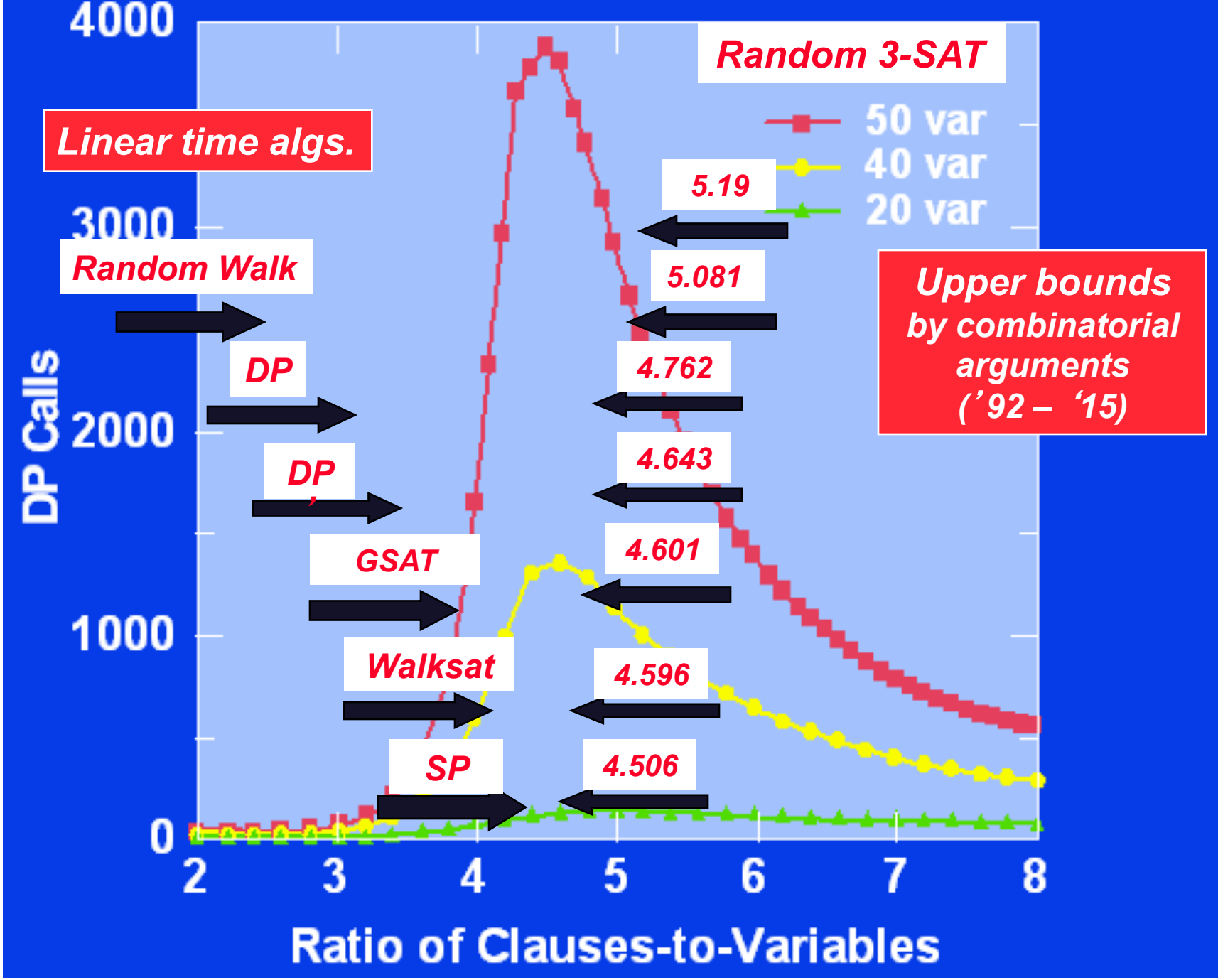
In P:
sorting, shortest path, …

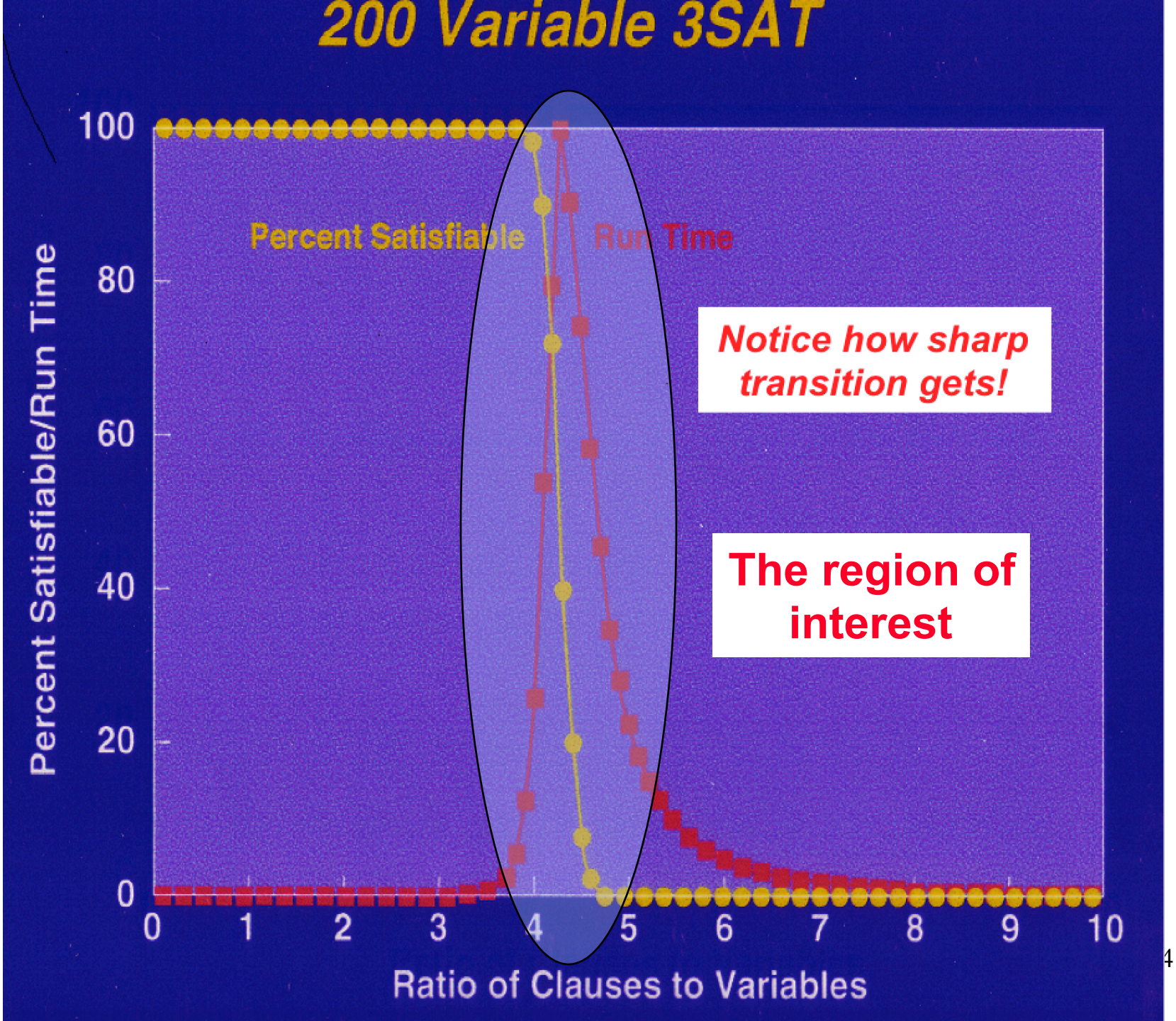


Note: widely believed hierarchy; know P≠EXP for sure



Mitchell, Selman, and Levesque '92


Random 3-SAT
50 var
40 var
20 var
Linear time algs.
Random Walk
DP
DP'
GSAT
Walksat
SP
5.19
5.081
4.762
4.643
4.601
4.596
4.506
Upper bounds by combinatorial arguments ('92 – '15)
DP Calls
4000
3000
2000
1000
0
2
3
4
5
6
7
8
Ratio of Clauses-to-Variables


200 Variable 3SAT
Percent Satisfiable
Run Time
Notice how sharp transition gets!
The region of interest
Percent Satisfiable/Run Time
Ratio of Clauses to Variables
100
80
60
40
20
0
0
1
2
3
4
5
6
7
8
9
10

New types of algorithms for SAT. For example, local search methods (e.g. WalkSAT) and survey propagation (SP), an advanced form of belief propagation.

General insights into practical complexity:

I) Easy-hard-easy patterns and "critically constrained problems"

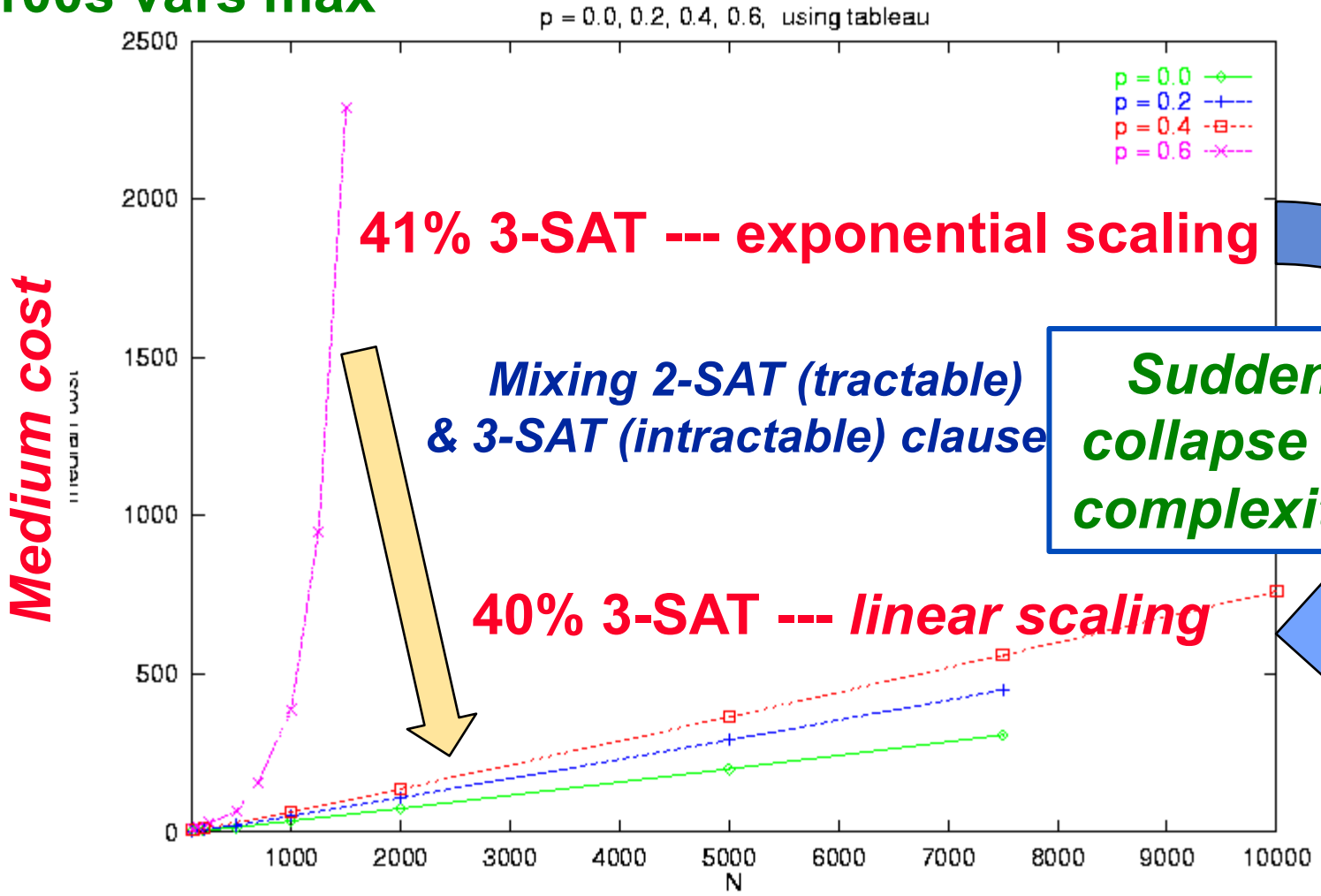
II) Surprise observation about mixing tractable and intractable structure. E.g. 2SAT and 3SAT. Partly explains the tremendous progress in SAT solving to follow.

Mixture of tractable and intractable structure

From $2^{O(N)}$ to O(N) scaling, if sufficient tractable structure is uncovered!

Few 100s vars max ☹

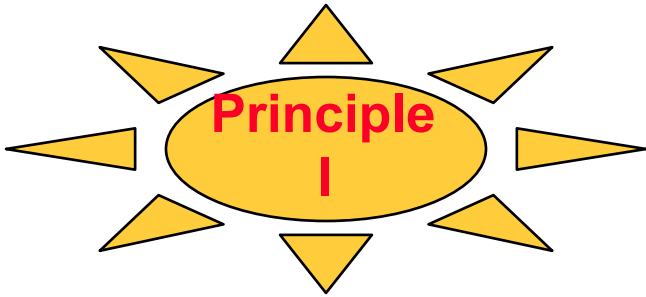
Millions of variables! ☺



Num variables

(Monasson, Selman et al. 99)

Scaling-Up Reasoning



Key to scalability in reasoning is uncovering substantial tractable substructure.

Mechanisms:

I) Constraint propagation (CSP) and unit-propagation (SAT).
Incomplete but highly efficient "sub-inference."

II) Clause learning ("no-good learning") adds derived constraints during search. Helps I).
Conflict Directed Clause Learning (CDCL) SAT solvers.

III) Randomization, restarts, and heuristic branching.
Backdoor variables.

Scaling-Up Reasoning, cont.

Techniques scale up reasoning from
a few hundred of variables max in the early 90s to
10+ million variable problems for current SAT solvers.

We can now revisit McCarthy's automated inference paradigm.

Contributors: [random order] Gomes, Kautz, Sabharwal, Ermon, Kroc, Levesque, Horvitz, Bessiere, Walsh, Gent, Zecchina, Mitchell, Leyton-Brown, Chen, Huang, Rintanen, Hoos, Achlioptas, Cheeseman, Kirkpatrick,Sandholm, Chayes, Brogs, Marques-Silva, Malik, O'Sullivan, Zhang, Lynce, Horvitz, Willams, van Harmelen, van Gelder, Sinz, Dilkina, Yexiang, Darwich, LeBras, Wei Wei, Freuder, Wilson, Kambhampati, Hoffmann, Bierre, Papadimitriou, Bacchus, Beame, Pitassi, McAllester, Weld, Geffner, Samulowitz, Sellmann, Seider, Clarke, Impagliazzo, Manya, Ansotague, Szeider, and others!!

Aside: A Taste of Problem Size

Consider a real world Boolean Satisfiability (SAT) problem, from formal verification.

The instance bmc-ibm-6.cnf, IBM LSU 1997:

```
p cnf !
−1 7 0
−1 6 0
−1 5 0
−1 −4 0
−1 3 0
−1 2 0
−1 −8 0
−9 15 0
−9 14 0
−9 13 0
−9 −12 0
−9 11 0
−9 10 0
−9 −16 0
−17 23 0
−17 22 0
```

I.e., ((not x_1) or x_7)
((not x_1) or x_6)
etc.

x_1, x_2, x_3, etc. our Boolean variables (set to True or False)

Set x_1 to False ??

10 pages later:

```
185 −9 0
185 −1 0
177 169 161 153 145 137 129 121 113 105 97
 89 81 73 65 57 49 41
 33 25 17 9 1 −185 0
186 −187 0
186 −188 0
```

…

I.e., (x_177 or x_169 or x_161 or x_153 …
x_33 or x_25 or x_17 or x_9 or x_1 or (not x_185))

clauses / constraints are getting more interesting…

Note x_1 …

4000 pages later:

10236 −10050 0
10236 −10051 0
10236 −10235 0
10008 10009 10010 10011 10012 10013 10014
10015 10016 10017 10018 10019 10020 10021
10022 10023 10024 10025 10026 10027 10028
10029 10030 10031 10032 10033 10034 10035
10036 10037 10086 10087 10088 10089 10090
10091 10092 10093 10094 10095 10096 10097
10098 10099 10100 10101 10102 10103 10104
10105 10106 10107 10108 −55 −54 53 −52 −51 50
10047 10048 10049 10050 10051 10235 −10236 0
10237 −10008 0
10237 −10009 0
10237 −10010 0

...

Finally, 15,000 pages later:

$-7\ 260\ 0$
$7\ -260\ 0$
$1072\ 1070\ 0$
$-15\ -14\ -13\ -12\ -11\ -10\ 0$
$-15\ -14\ -13\ -12\ -11\ 10\ 0$
$-15\ -14\ -13\ -12\ 11\ -10\ 0$
$-15\ -14\ -13\ -12\ 11\ 10\ 0$
$-7\ -6\ -5\ -4\ -3\ -2\ 0$
$-7\ -6\ -5\ -4\ -3\ 2\ 0$
$-7\ -6\ -5\ -4\ 3\ -2\ 0$
$-7\ -6\ -5\ -4\ 3\ 2\ 0$
$185\ 0$

Search space of truth assignments:

$$2^{50000} \approx 3.160699437 \cdot 10^{15051}$$

Current SAT solvers solve this instance in a few seconds!