Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior

Xavier Provot, 1995

Jeff Budsberg • April 12, 2005 • Cornell University

Early Work in Cloth

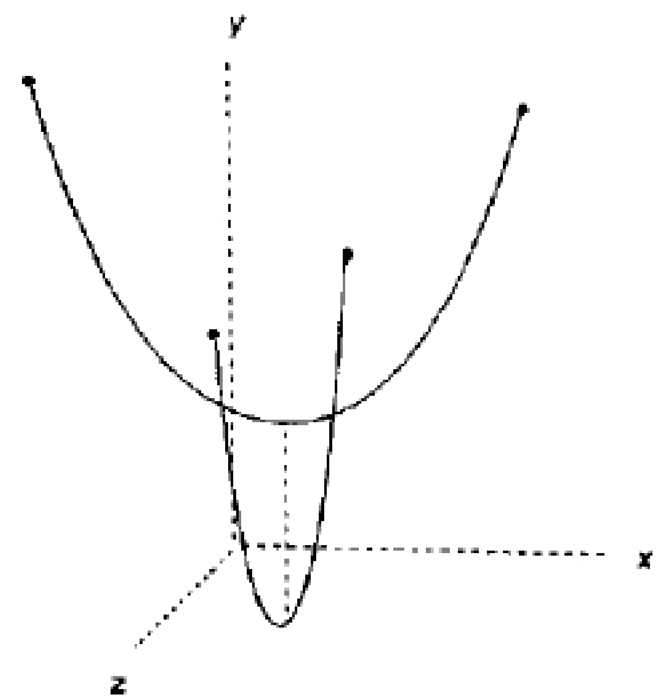
- Geometric models
  - Do not consider cloth's physical properties
  - focus on appearance (particularly folds and creases)
  - Jerry Weil, 1986
- Physical models
  - Various structural studies are done and cloth's intrinsic behavior is attempted to be simulated
  - C. Feynman, 1986
  - Demetri Terzopoulos et all, 1987

Early Work in Cloth

- Particle models
  - Explicitly represents the microstructure of woven cloth with interacting particles
  - David Breen et all, 1994

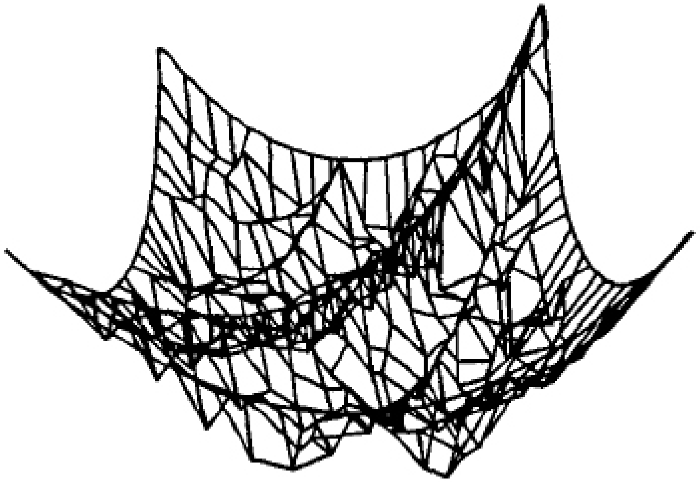
Jerry Weil

- Probably the first person to model cloth in any method whatsoever
- A cable under self-weight forms a catenary curve at equilibrium
- A cloth hanging from a discrete number of points can be described by a system of these curves

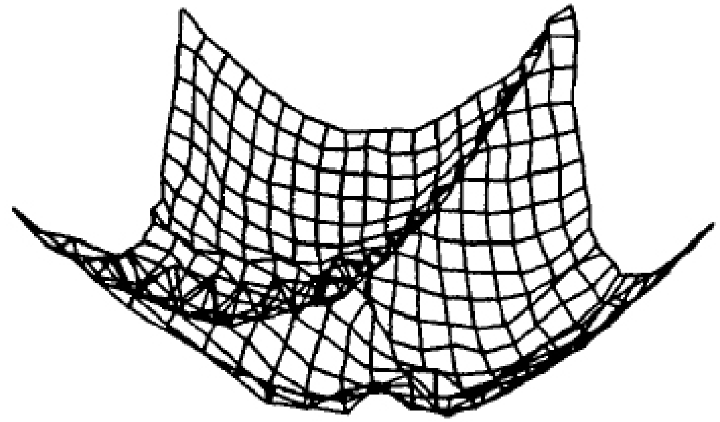


$$y = a \cosh(x/b)$$

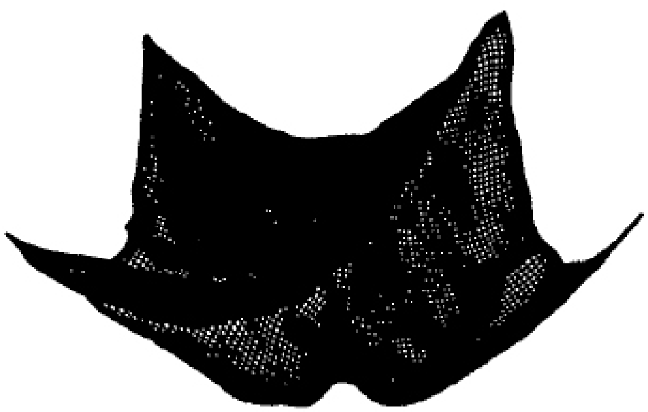
Jerry Weil



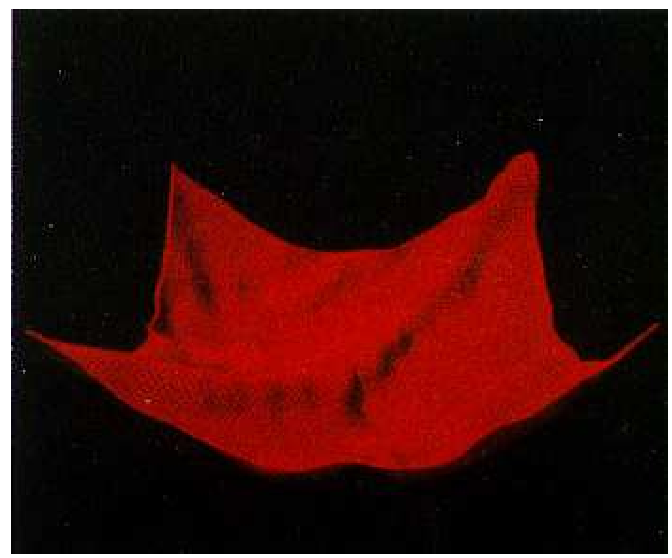
Surface Approximation



6 Iterations of Relaxation

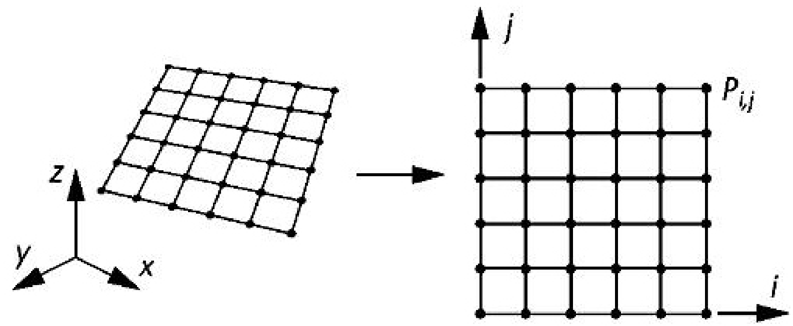


Spline Fit



C. Feynman

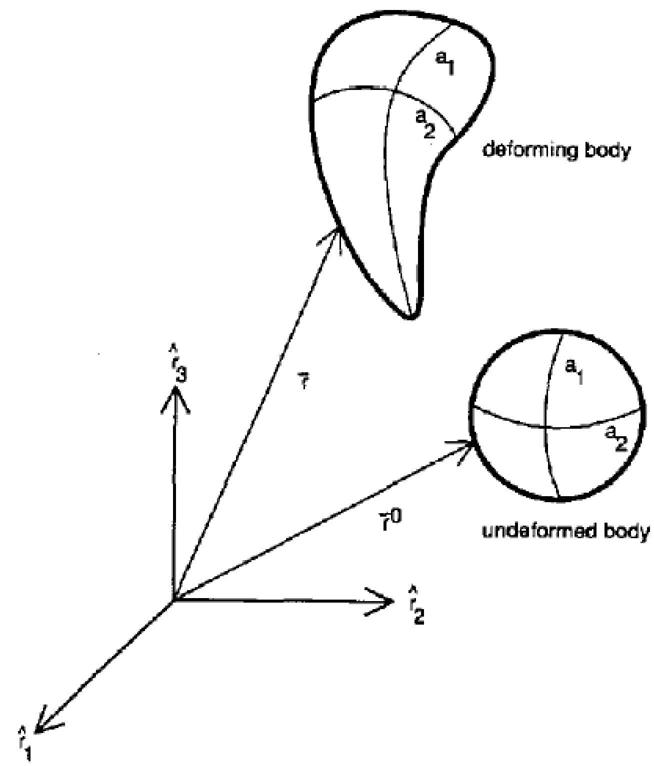
- Represented cloth in a 3D space by using a 2D grid
- The energy for each point is calculated in relation to its surrounding points
- The final position of cloth was derived based on the minimization of energy



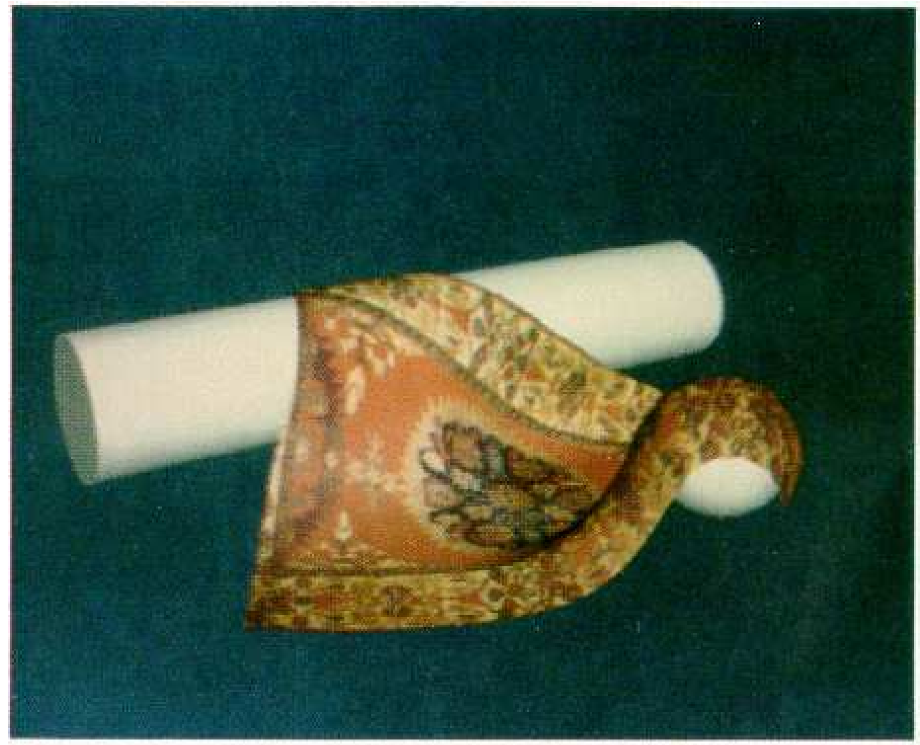
$$E(P_{i,j}) = k_s E_{elastic(i,j)} + k_b E_{bending(i,j)} + k_g E_{gravitational(i,j)}$$

Demetri Terzolpoulos

- Introduced a deformable model intended for generalized flexible objects
- Does not consider weave of the cloth, but only one internal elastic force
- Uses the Lagrange equation of motion to determine the equilibrium



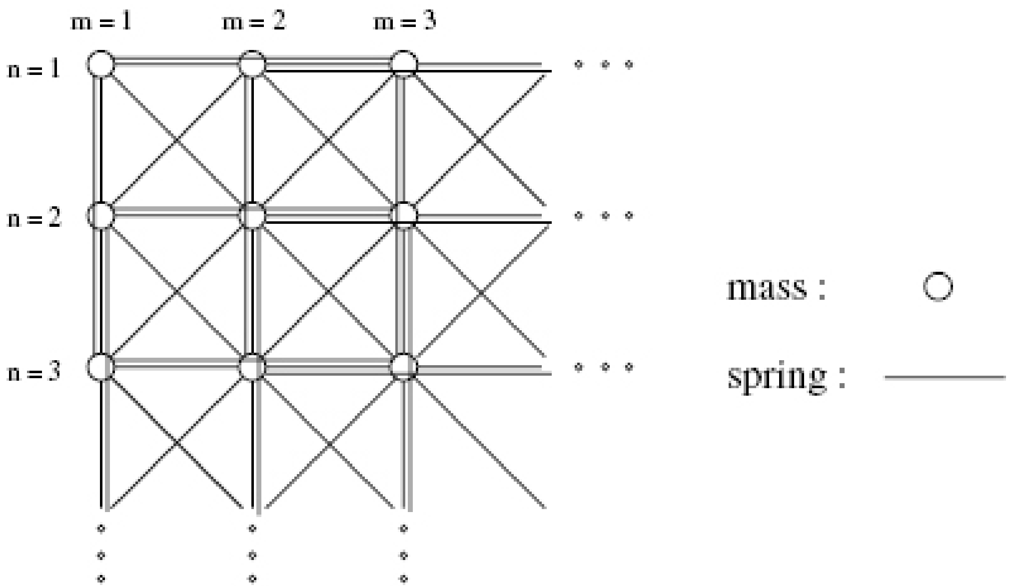
Demetri Terzolpoulos

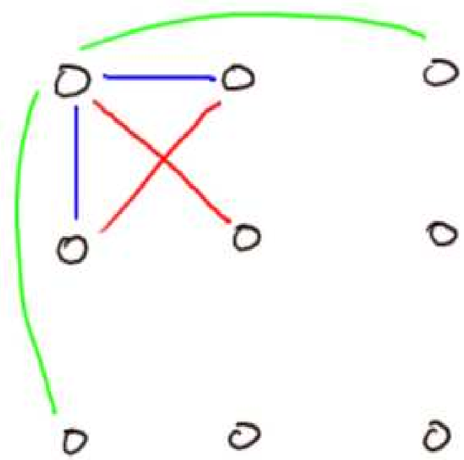


Motives

- Woven fabrics are far from having ideal elastic properties
- Physically-based, elastically-deformable models somewhat successful (“super-elastic” problem)
- Attempts at other methods:
  - Network of rigid rods of a fixed length (shearing, slow)
  - Particle system (static, slow)

Cloth Grid Mesh





Structural springs, resist stretching stresses

Sheer springs, resist sheering stresses

Flexion springs, resist bending stresses

Dynamics and Forces

- Once a mass-spring grid has been created, forces are applied to the nodes to generate an animation.
- The system is the mesh of $m \times n$ masses, each mass with position at time t given by $P_{i,j}(t)$
- The evolution of the system is governed by the fundamental law of dynamics:

$$F_{i,j} = \mu a_{i,j}$$

  where μ is the mass at point $P_{i,j}(t)$, and $a_{i,j}$ is the acceleration caused by the force $F_{i,j}$.
- $F_{i,j}$ can be divided into internal and external forces

Internal Force

- **Tensions of interconnected springs**

Which are described by Hooke's Law:

$$F = k \bullet u$$

where F is the applied force, u is the deformation (displacement from equilibrium) of the elastic body subjected to the force F, and k is the spring constant.

Internal Force

- In our case, this force is basically the sum of the change in point vectors multiplied by the spring stiffness for each neighbor of each point.

$$\mathbf{F}_{int}(P_{i,j}) = -\sum_{(k,l)\in\mathcal{R}} K_{i,j,k,l}\,[\mathbf{l}_{i,j,k,l} - l^0_{i,j,k,l}\,\frac{\mathbf{l}_{i,j,k,l}}{\|\mathbf{l}_{i,j,k,l}\|}]$$

- $\mathcal{R}$ is the set regrouping all couples (k,l) such as $P_{k,l}$ is linked by a spring to $P_{i,j}$,
- $\mathbf{l}_{i,j,k,l} = \overrightarrow{P_{i,j}P_{k,l}}$,
- $l^0_{i,j,k,l}$ is the natural length of the spring linking $P_{i,j}$ and $P_{k,l}$,
- $K_{i,j,k,l}$ is the stiffness of the spring linking $P_{i,j}$ and $P_{k,l}$.

External forces

- **Force of gravity**

$$F_{gr}(P_{i,j}) = \mu g$$

where g is the acceleration of gravity

- **Viscous damping**

$$F_{dis}(P_{i,j}) = -C_{dis}v_{i,j}$$

where C_{dis} is the damping coefficient and $v_{i,j}$ is the velocity at point $P_{i,j}$.

External forces

- **Viscous fluid (wind)**

$$F_{vi}(P_{i,j}) = C_{vi}\,[n_{i,j} \cdot (u_{fluid} - v_{i,j})]\, n_{i,j}$$

where u_{fluid} is a viscous fluid with uniform velocity, $v_{i,j}$ is the velocity at point $P_{i,j}$, $n_{i,j}$ is the unit normal at $P_{i,j}$, and C_{vi} is the viscosity constant

- The **net force** acting on any node in the mass-spring model is the **sum of the above forces** for that node.

Integration

•To generate animation of cloth, it is necessary to compute the location of the nodes for a series of time steps.

•Provot uses a simple Euler method to approximate the fundamental equation of dynamics.

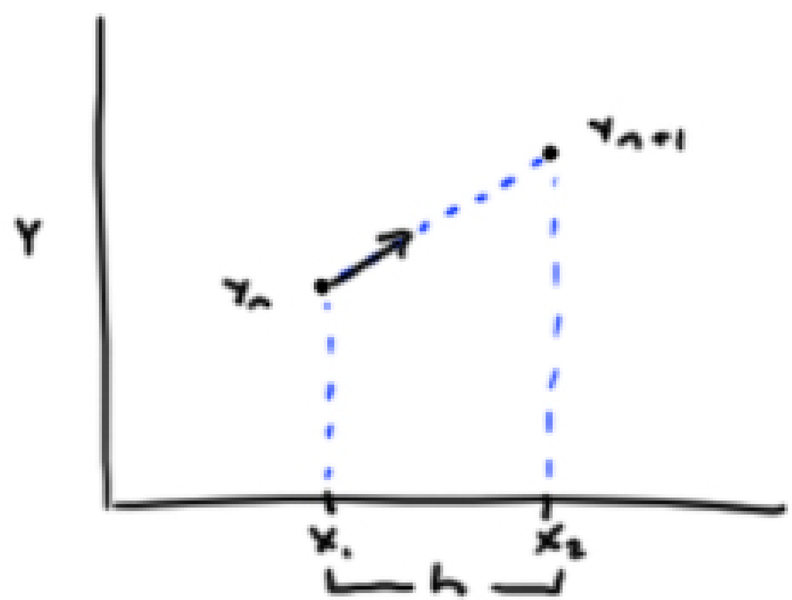
$$\mathbf{a}_{i,j}(t+\Delta t) = \frac{1}{\mu}\mathbf{F}_{i,j}(t)$$

$$\mathbf{v}_{i,j}(t+\Delta t) = \mathbf{v}_{i,j}(t) + \Delta t\, \mathbf{a}_{i,j}(t+\Delta t)$$

$$P_{i,j}(t+\Delta t) = P_{i,j}(t) + \Delta t\, \mathbf{v}_{i,j}(t+\Delta t)$$

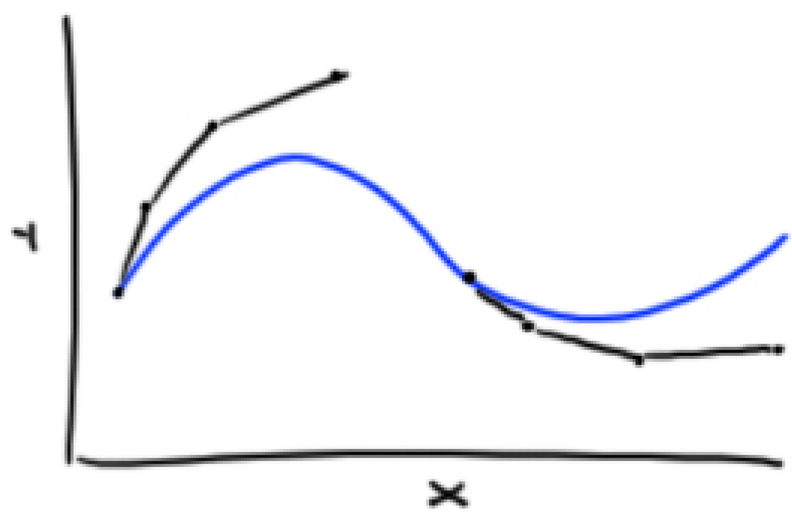
Forward Euler

- In this method, the position of the nodes in the next time step are computed using only past information.



Forward Euler Error

- Explicit integration has numerous problems including instability at large time steps and slow propagation of the effects of forces over the cloth material.



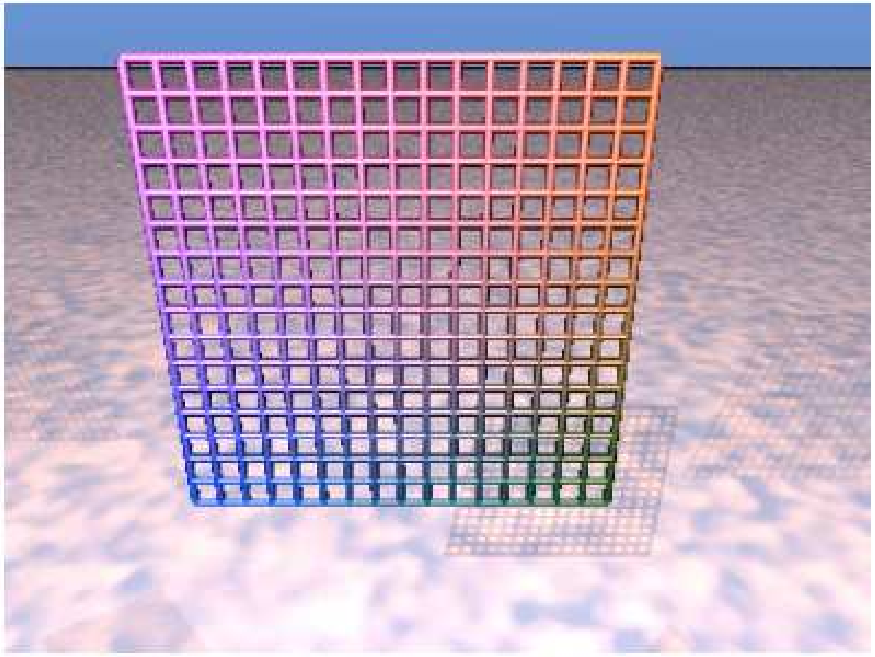
Dynamic Inverse Procedures

- Some cases where cloth movement is not entirely caused by analytically computed forces (contact problems)
- So far, we can compute displacement of a point due to a force applied to it, but we can solve the inverse problem for hanging points
- A similar procedure can be used to deal with object collisions and self-intersection, though not covered in this paper (Provot 97)

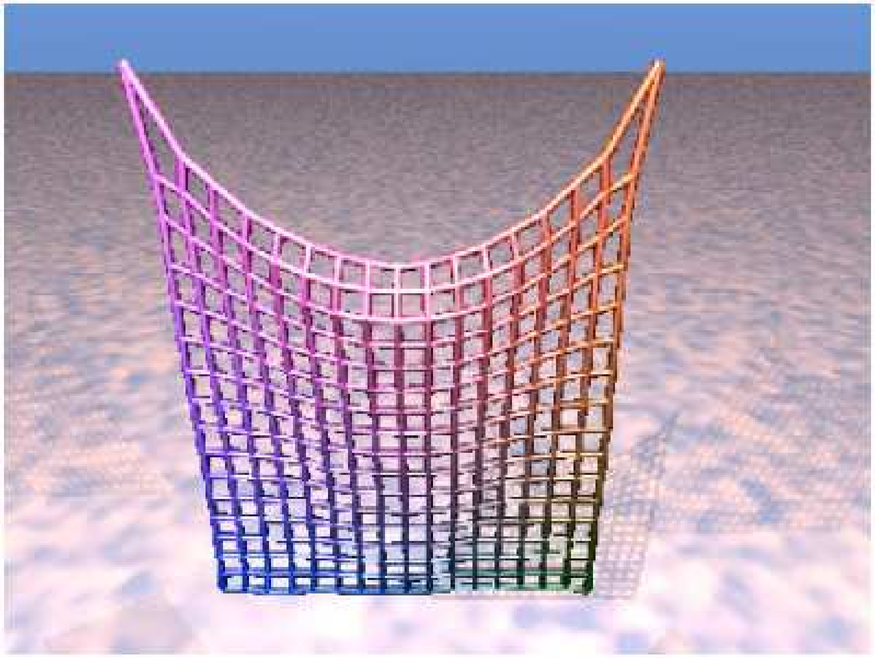
Collision Detection

- Collisions of two types
  - Point-triangle collision
  - Edge-edge collision
- At a time where a collision is detected, a physics-based response is calculated
- Accurate, but limited in that all nodes are assumed to have constant velocity
- Successful in simulating draping
- Problems with sliding contact and jittering

The “Super-Elastic” Effect



Initial position



After 200 iterations

The “Super-Elastic” Effect

- Case study: subject to gravity, but no wind
- Concentration of local deformations
- Deformation rate[1] decreases very rapidly
- **Real-world problem:** such a deformation never occurs since real woven fabrics have non-linear elasticity (and tear when high loads are applied)

1) The deformation rate is defined as $\tau=(l-l_0)/l_0$

where l_0 is the natural spring length and l is its length at any time t

Increasing Stiffness

- Stiffer springs should lower the deformation rate
- For a given time step Δt and mass μ, there is a critical stiffness value K_c above which the numerical resolution of the system is divergent
- Thus, the maximum Δt is equal to the natural period of a simple harmonic oscillator (mass on a spring):

$$T_0 \approx \pi\sqrt{\frac{\mu}{K}}$$

$$\Longrightarrow K_c \approx m\frac{T_0^2}{\pi^2}$$

Increasing Stiffness

- If we want to increase stiffness, we have to decrease Δt below the new decreased value of T_0
- Need new method to avoid the super-elastic effect, without decreasing Δt

Constraints on Deformation Rates

- Assume that the direction of the elongated spring is correct, but limit it to a **critical deformation rate** (τ_c)

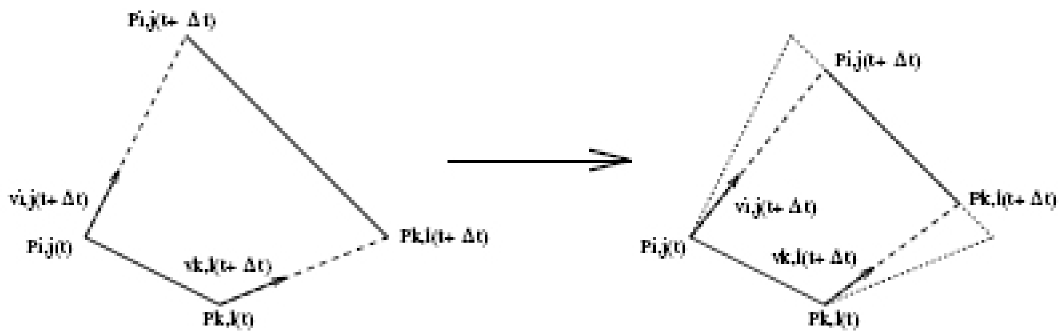
$$\textit{for}\ (\Delta t{+}{+})$$

$$\textit{Compute every}\ \tau$$

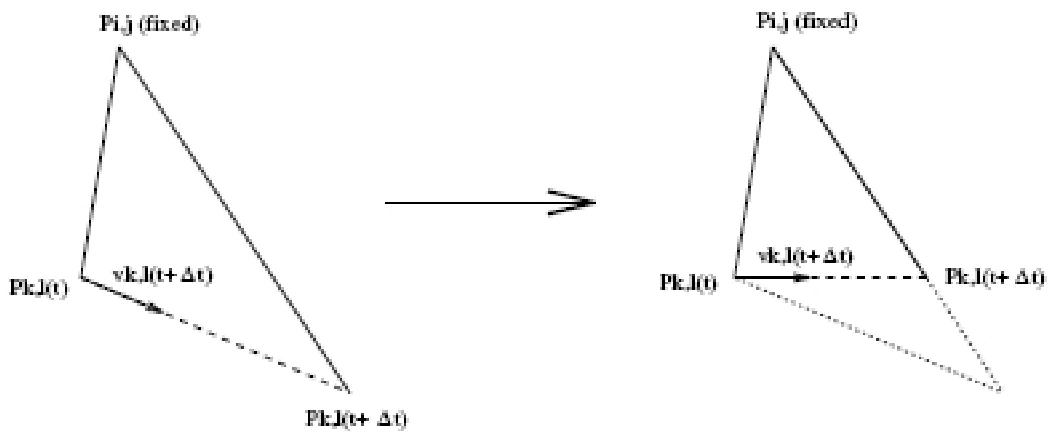
$$\textit{if}\ (\ \epsilon\ \tau\ \|\ \tau > \tau_c\ )$$

$$\tau = \tau_c$$

Constraints on Deformation Rates

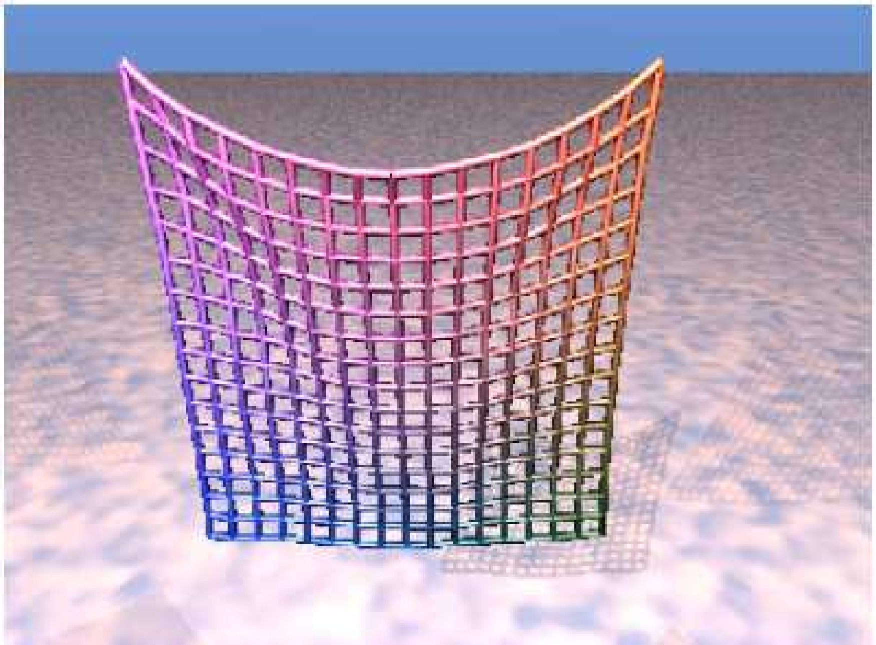


Adjustment of super-elongated spring linking two loose masses

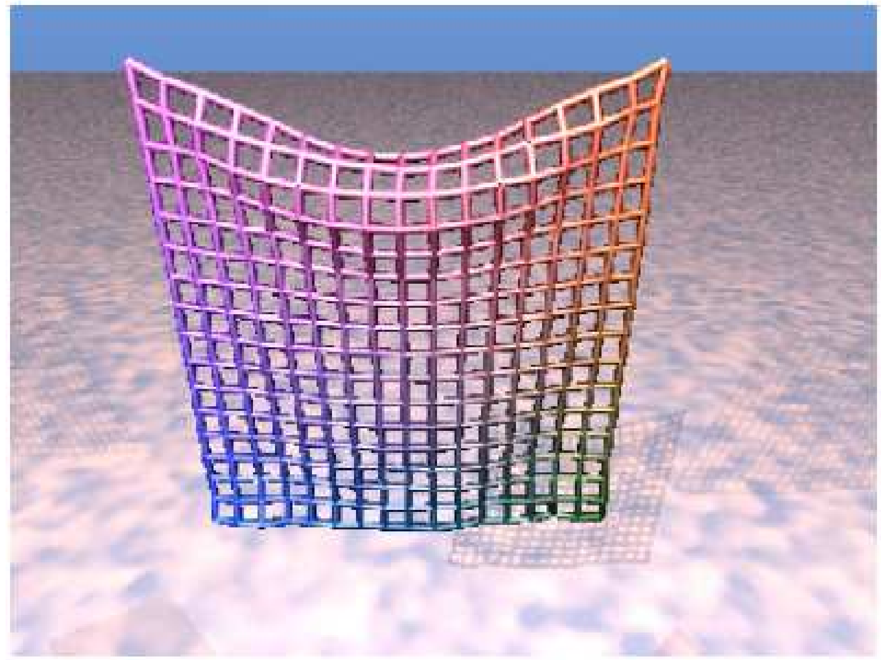


Adjustment of super-elongated spring linking a fixed and a loose mass

Hanging Sheet Results

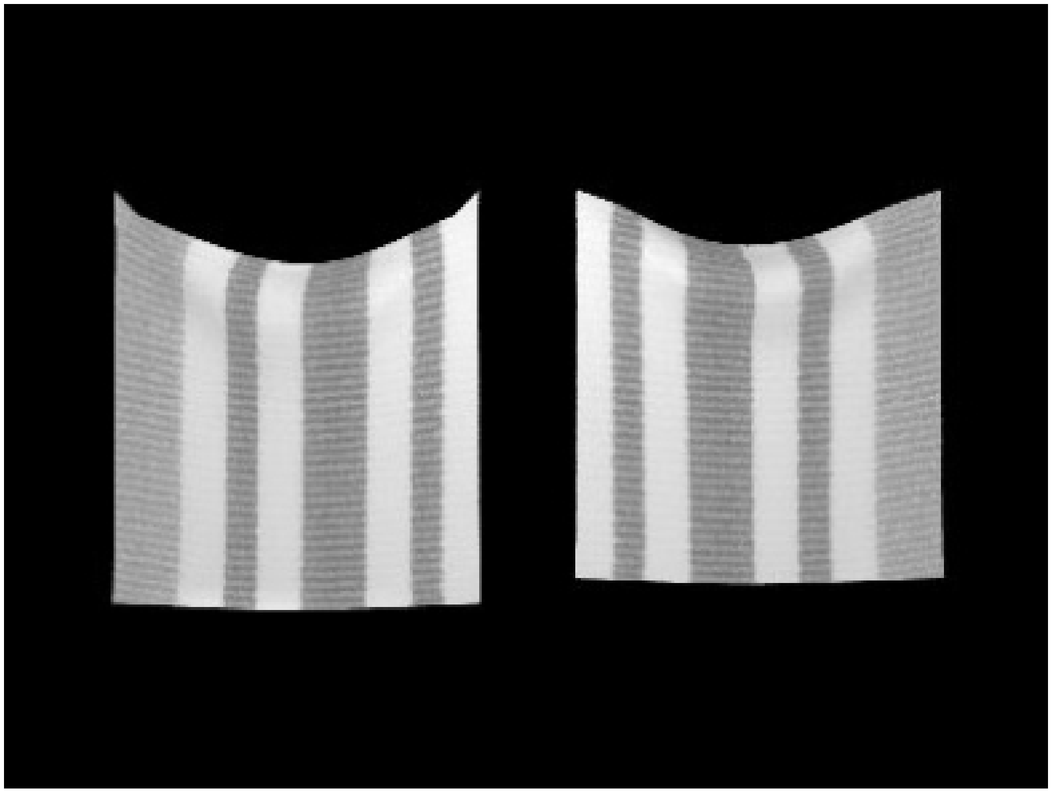


$r_{c\ (structural)} = 10\%$
$r_{c\ (flexion)} = 0\%$



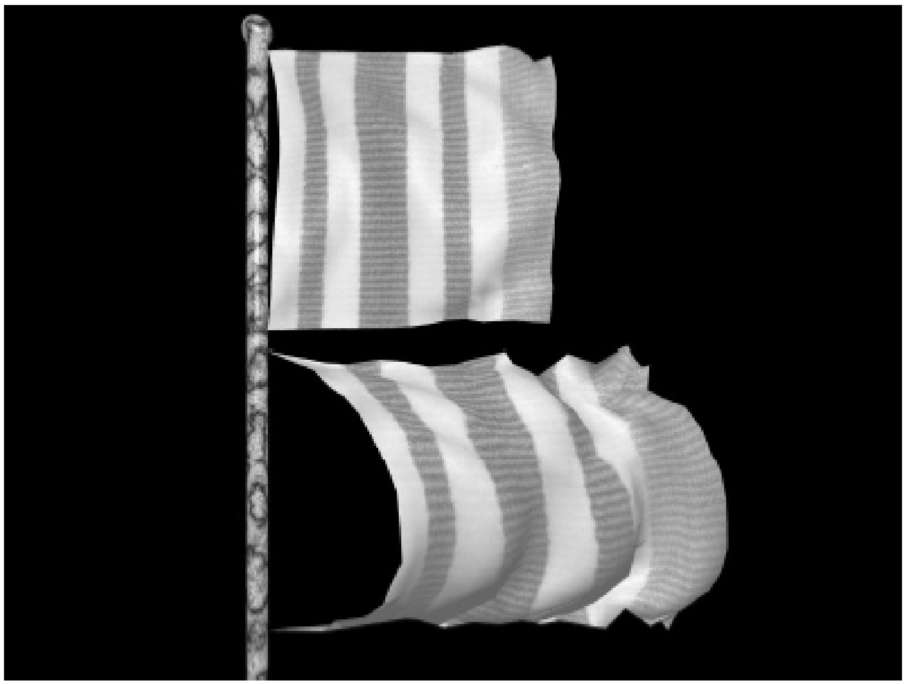
$r_{c\ (structural/shear)} = 10\%$
$r_{c\ (flexion)} = 0\%$

Hanging Sheet Comparison

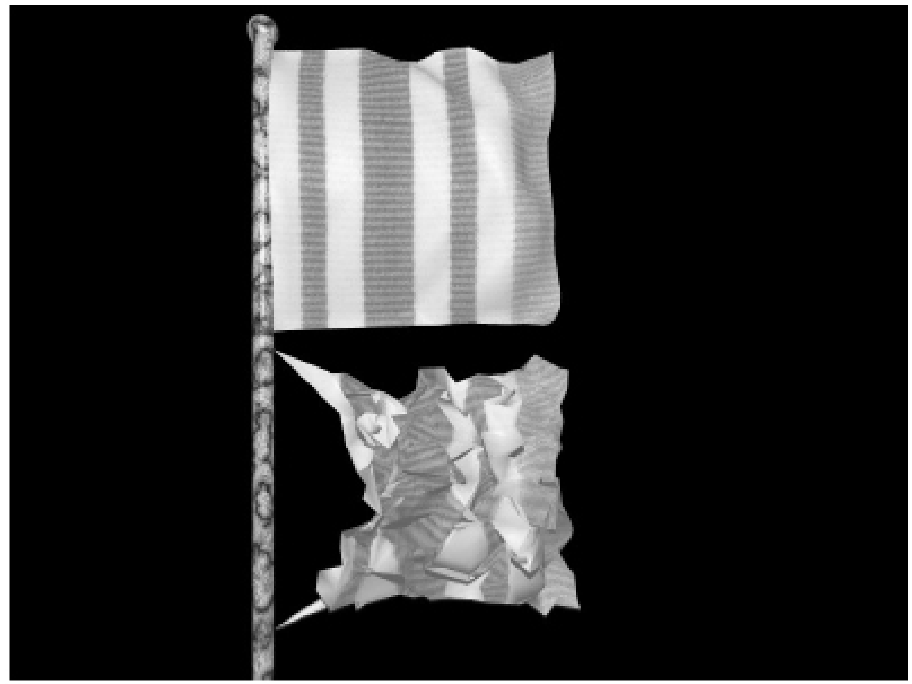


On the left: a stiff elastic model computed in 9 min.
On the right: new model computed in 1 min.

Flag in a Strong Wing

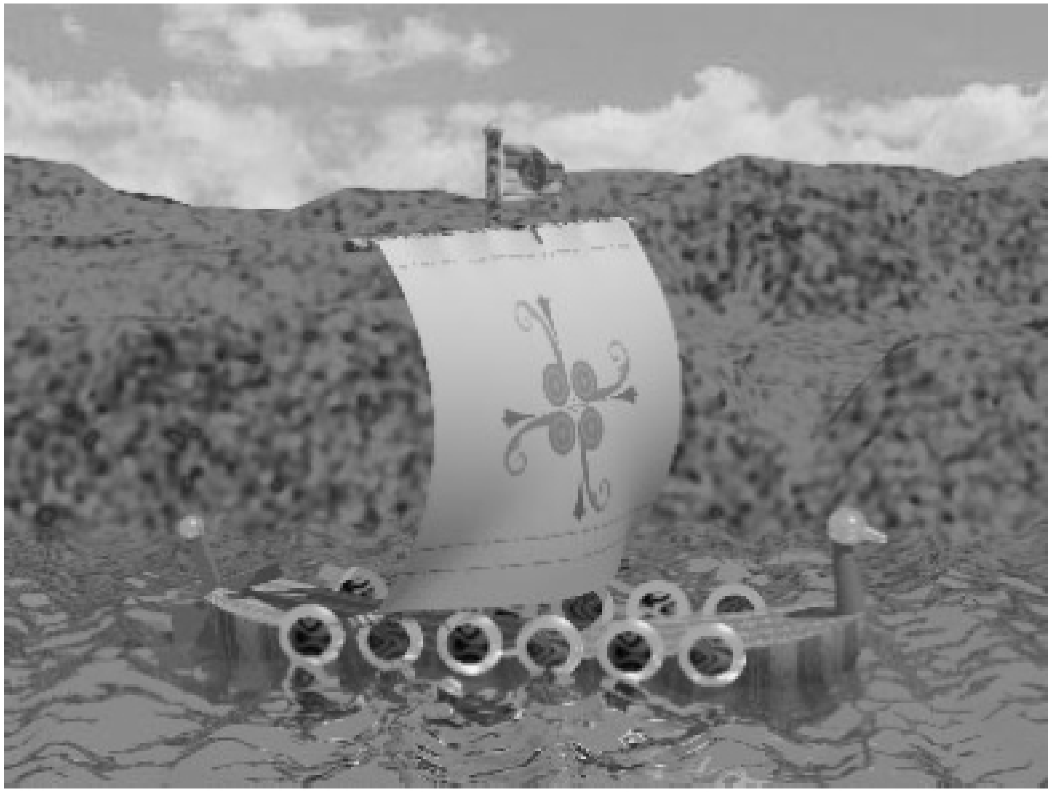


Semi-rigid flag and elastic
flag when stiffness is low



Semi-rigid flag remains stable
when stiffness is increased

Wind in a Sail



Hangs by 8 points on the upper rod and is ties to 2 points on the lower rod

Disadvantages

- Diverges from strictly physics-based simulation
- Dependent on the order in which the springs are examined
- Correcting one spring may overextend another
- Reiterative process does not always converge to a completely non-extended state

Advantages

- Produces realistic-looking output in most cases
- The time constant does not need to be reduced to match higher spring constants
- Can use an order of magnitude large time step
- 90% reduction in running time of the simulation according to the author
- Thus this model sacrifices a tolerable amount of accuracy for a dramatic speed improvement

Summary

- A physically-based model for animating cloth objects
- Derived from elastically-deforming models, but takes into account non-elastic properties of woven fabrics
- Cloth object approximated with a deformable surface network of masses and springs
- Dynamic inverse procedure to correct for unrealistic local deformation about the boundary conditions.