CS 6650 Computational Motion **Spring 2011, Prof. Doug James**
Homework #2: [Baraff and Witkin 1998] **Due Wed Feb 16 (in class)**

In [Baraff and Witkin 1998] there are various energy terms defined to model cloth forces associated with stretching, shearing, and bending deformations. Given a condition's m-vector $\mathbf{C}(\mathbf{x}) = (C_1, \ldots, C_m)^T$, they define a scalar energy contribution of stiffness k,

$$E_C(\mathbf{x}) = \frac{k}{2}\mathbf{C}(\mathbf{x})^T\,\mathbf{C}(\mathbf{x}) = \frac{k}{2}\sum_{l=1}^{m}(C_l(\mathbf{x}))^2\,.$$

where I will use sums over the m components to simplify interpretation. The resulting 3-vector force contributed to node i is

$$\mathbf{f}_i = -\frac{\partial E_C}{\partial \mathbf{x}_i} = -k\sum_{l=1}^{m}\frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_i}C_l(\mathbf{x}),$$

and the 3-by-3 derivative matrix (stiffness matrix) contribution can be written

$$\mathbf{K}_{ij} = \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j^T} = -\sum_{l=1}^{m}\left(\frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_i}\frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_j^T} + \frac{\partial^2 C_l(\mathbf{x})}{\partial \mathbf{x}_i \partial \mathbf{x}_j^T}C_l(\mathbf{x})\right).$$

In practice, one needs to derive the condition partial derivatives, $\frac{\partial C_l}{\partial \mathbf{x}_k}$ and $\frac{\partial^2 C_l}{\partial \mathbf{x}_i \partial \mathbf{x}_j^T}$ in order to evaluate the resulting force vector and stiffness matrix. In this assignment, you will do precisely that.

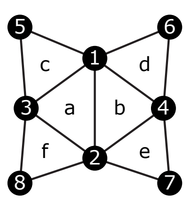
Please answer the following questions (simplify your hand-derived expressions as much as possible):

1. Show how to evaluate $\mathbf{f}_i(\mathbf{x})$ for stretch forces.

2. Show how to evaluate $\mathbf{f}_i(\mathbf{x})$ for shear forces.

3. Show how to evaluate $\mathbf{f}_i(\mathbf{x})$ for bend forces.

4. Show how to evaluate $\mathbf{K}_{ij}(\mathbf{x})$ for shear forces.

5. 

Given a simple 8-vertex mesh consisting of a 6-triangle butterfly, describe the sparsity patterns of $\mathbf{K}^{stretch}$, $\mathbf{K}^{shear}$, $\mathbf{K}^{bend}$ and the resulting

$$\mathbf{K} = \mathbf{K}^{stretch} + \mathbf{K}^{shear} + \mathbf{K}^{bend}.$$

For simplicity, only consider the block-level sparsity pattern–report 8-by-8 sparsity patterns.

References

David Baraff and Andrew P. Witkin. Large Steps in Cloth Simulation. In *Proceedings of SIGGRAPH 98*, Computer Graphics Proceedings, Annual Conference Series, pages 43-54, July 1998.