

In [Baraff and Witkin 1998] there are various energy terms defined to model cloth forces associated with stretching, shearing, and bending deformations. Given a condition's m -vector $\mathbf{C}(\mathbf{x}) = (C_1, \dots, C_m)^T$, they define a scalar energy contribution of stiffness k ,

$$E_C(\mathbf{x}) = \frac{k}{2} \mathbf{C}(\mathbf{x})^T \mathbf{C}(\mathbf{x}) = \frac{k}{2} \sum_{l=1}^m (C_l(\mathbf{x}))^2.$$

where I will use sums over the m components to simplify interpretation. The resulting 3-vector force contributed to node i is

$$\mathbf{f}_i = -\frac{\partial E_C}{\partial \mathbf{x}_i} = -k \sum_{l=1}^m \frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_i} C_l(\mathbf{x}),$$

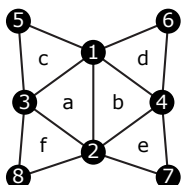
and the 3-by-3 derivative matrix (stiffness matrix) contribution can be written

$$\mathbf{K}_{ij} = \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j^T} = -\sum_{l=1}^m \left(\frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_i} \frac{\partial C_l(\mathbf{x})}{\partial \mathbf{x}_j^T} + \frac{\partial^2 C_l(\mathbf{x})}{\partial \mathbf{x}_i \partial \mathbf{x}_j^T} C_l(\mathbf{x}) \right).$$

In practice, one needs to derive the condition partial derivatives, $\frac{\partial C_l}{\partial \mathbf{x}_k}$ and $\frac{\partial^2 C_l}{\partial \mathbf{x}_i \partial \mathbf{x}_j^T}$ in order to evaluate the resulting force vector and stiffness matrix. In this assignment, you will do precisely that.

Please answer the following questions (simplify your hand-derived expressions as much as possible):

1. Show how to evaluate \mathbf{f}_i for stretch forces.
2. Show how to evaluate \mathbf{f}_i for shear forces.
3. Show how to evaluate \mathbf{f}_i for bend forces.
4. Show how to evaluate \mathbf{K}_{ij} for shear forces.



5. Given a simple 8-vertex mesh consisting of a 6-triangle butterfly, describe the sparsity patterns of $\mathbf{K}^{stretch}$, \mathbf{K}^{shear} , \mathbf{K}^{bend} and the resulting

$$\mathbf{K} = \mathbf{K}^{stretch} + \mathbf{K}^{shear} + \mathbf{K}^{bend}.$$

For simplicity, only consider the block-level sparsity pattern—report 8-by-8 sparsity patterns.

References

David Baraff and Andrew P. Witkin. Large Steps in Cloth Simulation. In *Proceedings of SIGGRAPH 98*, Computer Graphics Proceedings, Annual Conference Series, pages 43-54, July 1998.