Lecture 4: Radiometry and Rendering Equation

Chapter 2 in Advanced GI

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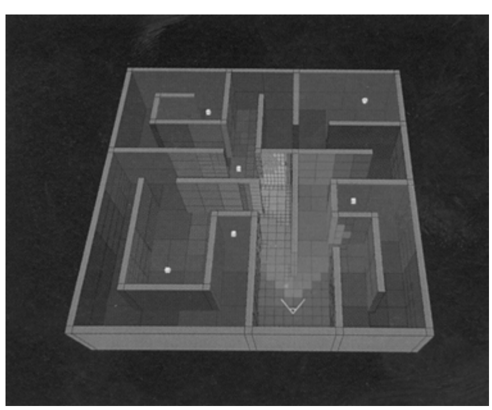
Radiosity+Importance

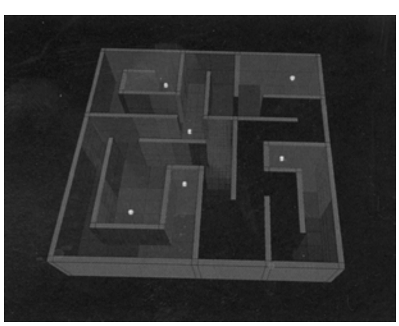
- Radiosity+Importance: Bidirectional



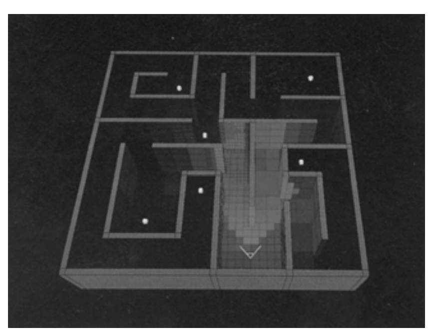
Importance Radiosity (IR)

- Motivation
  - $O(k^2 + n)$ is too slow
  - HR oversolves globally, undersolves locally
- Insight: Exploit view dependence
- Importance: Direct or indirect contribution of patch to image from this view point

Radiosity, Importance



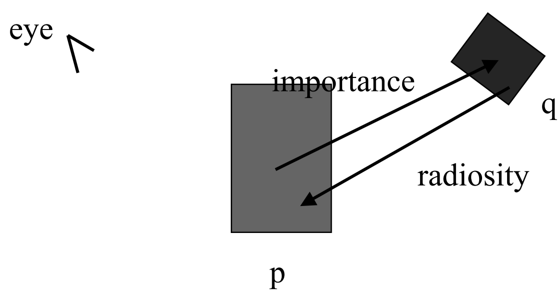
Radiosity: Forward



Importance: Backward

IR Intuition

- Importance: adjoint formulation of radiosity



IR Algorithm

- Solves for dual system simultaneously
- Importance is shot by treating eye as light source
- Importance R_i proportional A_i on image

Radiosity Equation

- Radiosity for each polygon i

$$\forall i : B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \rightarrow j)$$

- Linear system
  – B_i : radiosity of patch i (unknown)
  – $B_{e,i}$: emission of patch i (known)
  – ρ_l : reflectivity of patch i (known)
  – $F(i \rightarrow j)$: form-factor (coefficients of matrix)

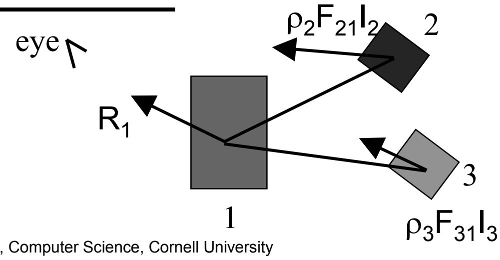
IR Intuition

- $I_1 = R_1 + \rho_2 F_{21} I_2 + \rho_3 F_{31} I_3 + \ldots$

Contribution of 1 through 2

Direct contribution of 1

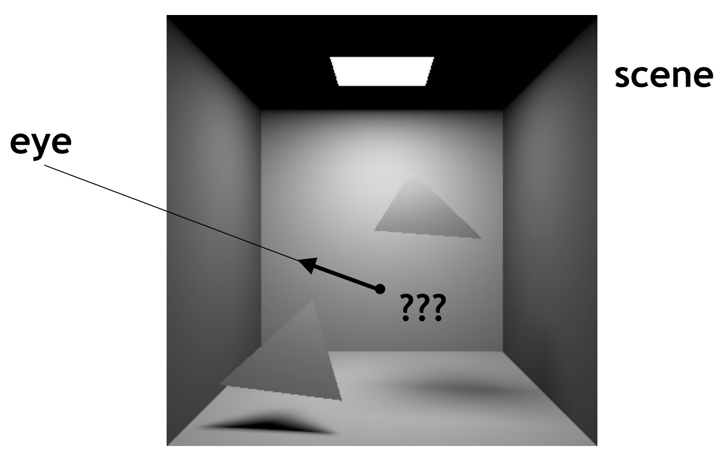
Contribution of 1 to eye



Importance Radiosity

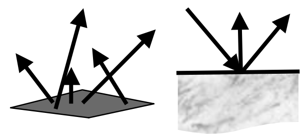
- Elegant formulation of bidirectional propagation
  - Replaces ad-hoc solutions
- IR restricted to one viewpoint
  - Need to unmesh as viewpoint moves

Motivation

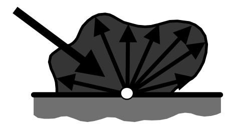


What is the behavior of light?

- Physics of light



- Radiometry
- Material properties



- **Rendering Equation**

Models of Light

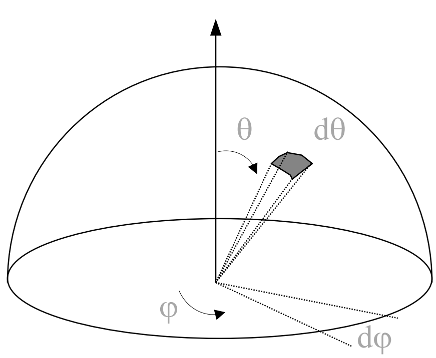
- Geometric Optics
  - Emission
  - Reflection / Refraction
  - Absorption
- Simplest model
- Size of objects > wavelength of light

Radiometry

- Radiometry: measurement of light energy
- Defines relation between
  - Power
  - Energy
  - Radiance
  - Radiosity

Digression: Hemispheres

- Hemisphere = two-dimensional surface
- Direction = point on (unit) sphere

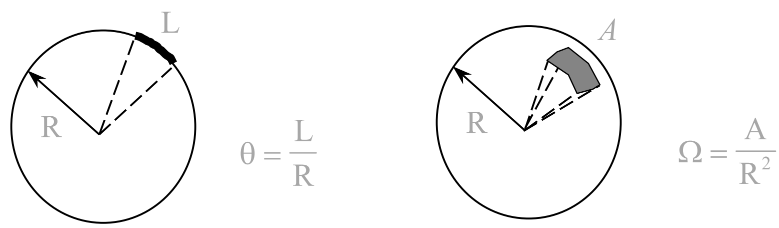


$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

Digression: Solid angles

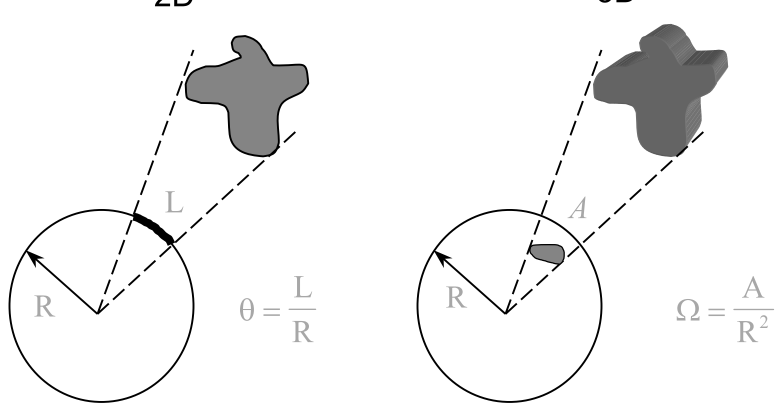
2D 3D



Full circle = 2π radians Full sphere = 4π steradians

Digression: Solid angles

2D 3D



Full circle = 2π radians Full sphere = 4π steradians

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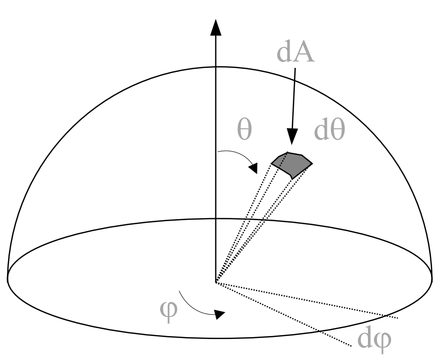
Digression: Solid angle

- Full sphere = 4π steradian = 12.566 sr
- Dodecahedron = 12-sided regular polyhedron; 1 face = 1 sr

Hemispherical coordinates

- Direction = point on (unit) sphere



$$dA = (r \sin\theta d\varphi)(r d\theta)$$

Hemispherical coordinates

- Defined a measure over hemisphere
- $d\omega$ = direction vector
- Differential solid angle

$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\varphi$$

Hemispherical integration

- Area of hemisphere:

Hemispherical integration

- Area of hemisphere:

$$\int_{\Omega_x} d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta$$

$$= \int_0^{2\pi} d\varphi \left[-\cos\theta\right]_0^{\pi/2}$$

$$= \int_0^{2\pi} d\varphi$$

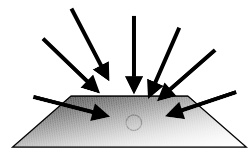
$$= 2\pi$$

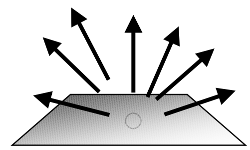
Power

- Energy: Symbol: *Q*; unit: Joules
- Power: Energy per unit time (*dQ*/*dt*)
  - Aka. "radiant flux" in this context
- Symbol: *P* or Φ; unit: Watts (Joules / sec)
  - Photons per second
  - All further quantities are derivatives of *P* (flux densities)

Irradiance

- Power per unit area (*dP*/*dA*)
  - That is, area density of power
  - It is defined with respect to a surface
- Symbol: *E*; unit: W / m^2
  - Measurable as power on a small-area detector
  - Area power density exiting a surface is called *radiant exitance* (*M*) or *radiosity* (*B*) but has the same units

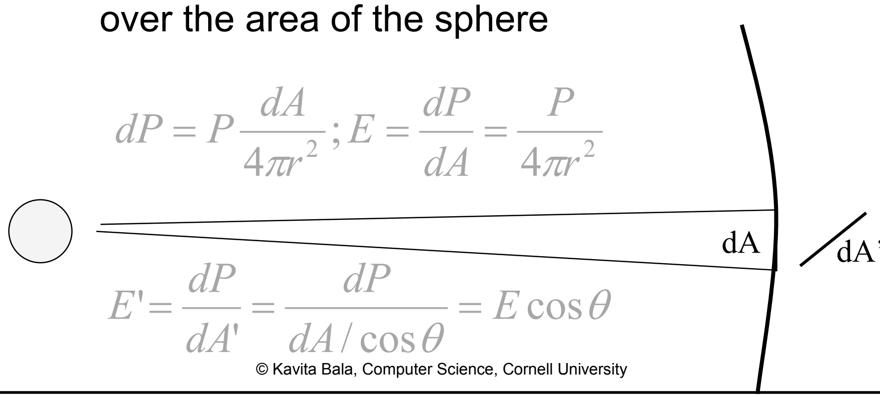




Irradiance example

- Uniform point source illuminates small surface *dA* from distance *r*
  - Think of it as a piece of a sphere
  - Power *P* is uniformly spread over the area of the sphere

$$dP = P\frac{dA}{4\pi r^2}; E = \frac{dP}{dA} = \frac{P}{4\pi r^2}$$

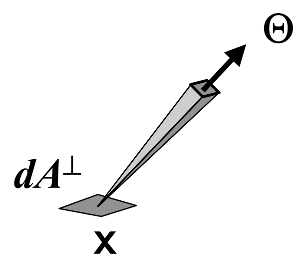


$$E' = \frac{dP}{dA'} = \frac{dP}{dA/\cos\theta} = E\cos\theta$$

Radiance

- Radiance is radiant energy at x in direction θ: 5D function
  - $L(x \to \Theta)$: Power
    - per unit projected surface area
    - per unit solid angle

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$



  - units: Watt / m^2.sr

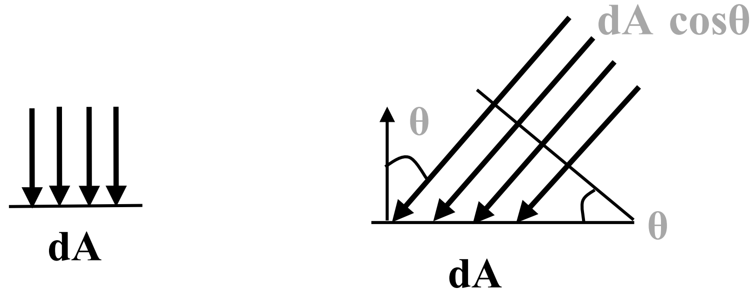
Radiance

- Power per unit (solid angle times area)
  - Counts photons that (a) go through a little area around *x* perpendicular to Θ *and* (b) are traveling in directions that fall in a little solid angle around Θ
  - Irradiance per unit solid angle
  - A 2nd derivative of *P*

Radiance: Projected area

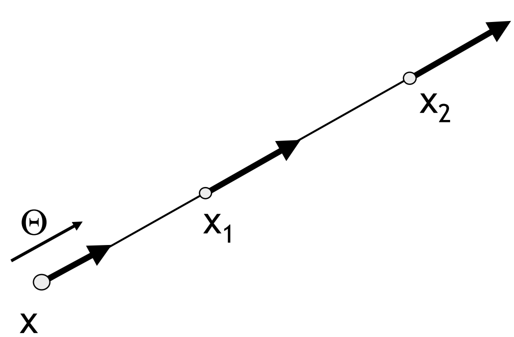
- $L(x \to \Theta) = \dfrac{d^2P}{dA^{\perp} d\omega_{\Theta}}$

- Why per unit projected surface area?

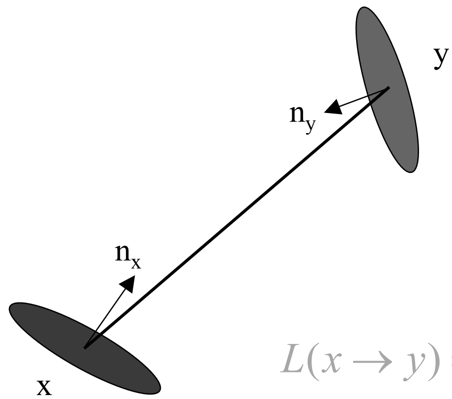


Why is radiance important?

- Invariant along a straight line (in vacuum)



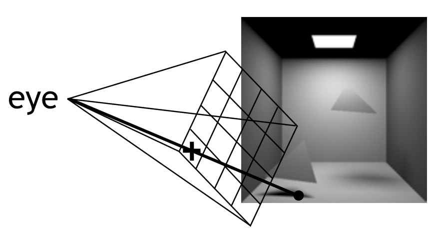
Invariance of Radiance



$$L(x \rightarrow y) = L(y \leftarrow x)$$

Why is radiance important?

- Response of a sensor (camera, human eye) is proportional to radiance



- Pixel values in image proportional to radiance received from that direction

Wavelength Dependence

- Each particle has a wavelength $E = \frac{h}{\lambda}$
- All radiometric quantities depend on wavelength
- Spectral radiance: $L(x \to \Theta, \lambda)$
- Radiance: $L(x \to \Theta) = \int L(x \to \Theta, \lambda) d\lambda$

Relationships

- Radiance is the fundamental quantity

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

- Power:

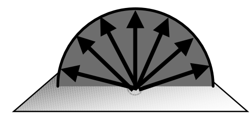
$$P = \int_{Area} \int_{\substack{Solid\\Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

- Radiosity:

$$B = \int_{\substack{Solid\\Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere



$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_\Theta}$$

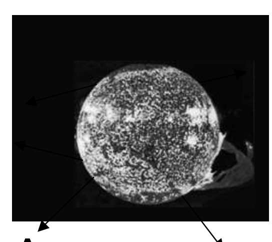
$$P = \int_{Area} \int_{\substack{Solid\\Angle}} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_\Theta \cdot dA$$

$$= L \int_{Area} dA \int_{\substack{Solid\\Angle}} \cos\theta \cdot d\omega_\Theta$$

$$= L \cdot Area \cdot \pi$$

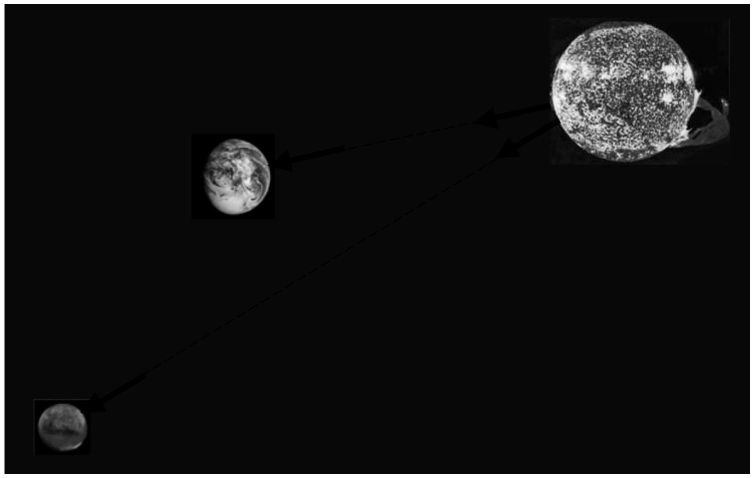
Sun Example: radiance

- Power: 3.91×10^{26} W
- Surface Area: 6.07×10^{18} m^2



- Power = Radiance.Surface Area.π
- Radiance = Power/(Surface Area.π)
- Radiance = 2.05×10^7 W/ m^2.sr

Sun Example

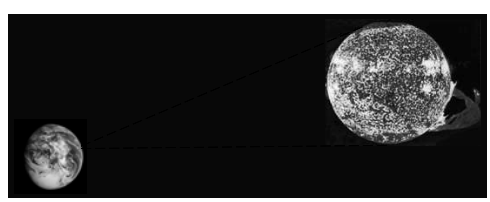


Same radiance on Earth and Mars?

Sun Example: Power on Earth

- Power reaching earth on a $1m^2$ square:

$$P = L \underbrace{\int}_{Area} dA \underbrace{\int}_{Solid\ Angle} \cos\theta \cdot d\omega_\Theta$$

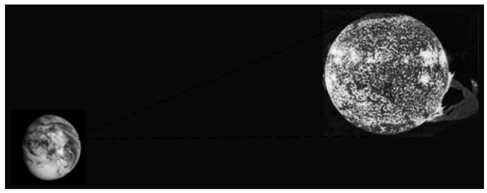


- Assume $\cos\theta = 1$ (sun in zenith)

$$P = L \underbrace{\int}_{Area} dA \underbrace{\int}_{Solid\ Angle} d\omega_\Theta$$

Sun Example: Power on Earth

Power = Radiance.Area.Solid Angle

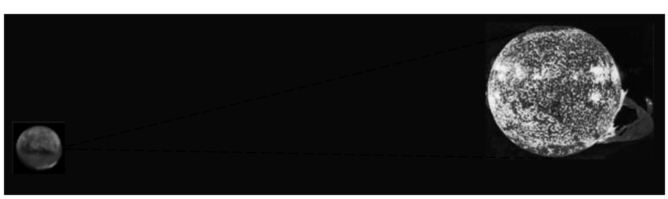


Solid Angle = Projected Area$_{Sun}$/(distance$_{earth_sun}$)^2
= $6.7\ 10^{-5}$ sr

P = (2.05×10^7 W/ m^2.sr) x (1 m^2) x ($6.7\ 10^{-5}$ sr)
= 1373.5 Watt

Sun Example: Power on Mars

Power = Radiance.Area.Solid Angle



Solid Angle = Projected Area$_{Sun}$/(distance$_{mars_sun}$)^2
= $2.92\ 10^{-5}$ sr

P = (2.05×10^7 W/ m^2.sr) x (1 m^2) x ($2.92\ 10^{-5}$ sr)
= 598.6 Watt