Lecture 3: Radiometry

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Information

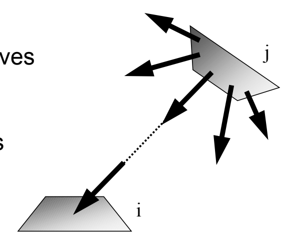
- Office Hours
  - Wed: 2-3 Upson 5142
- Web-page
- www.cs.cornell.edu/courses/cs665/2004fa/
  - Tentative schedule
  - Homeworks, lecture notes, will be on-line
  - Check for updates and announcements

Radiosity Algorithm

- Subdivide scene in polygons
  - mesh that determines final solution
- Compute Form Factors
  - transfer of energy between polygons
- Solve linear system
  - results in power (color) per polygon
- Pick a viewpoint and display
  - loop

Form Factor

- $F_{j \to i}$ = the fraction of power emitted by j, which is received by i
- Area
  - if i is smaller, it receives less power
- Orientation
  - if i faces j, it receives more power
- Distance
  - if i is further away, it receives less power



Form Factors - how to compute?

- Closed Form
  - Analytic
- Hemicube
- Monte-Carlo

How To Solve Linear System

- Matrix Inversion
- Gathering methods
  - Jacobi iteration
  - Gauss-Seidel
- Shooting
  - Southwell iteration
  - Improved Southwell iteration

Matrix Inversion

$$\begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_n \end{bmatrix} = \begin{bmatrix} 1-\rho_1 F_{1\to 1} & -\rho_1 F_{1\to 2} & \dots & -\rho_1 F_{1\to n} \\ -\rho_2 F_{2\to 1} & 1-\rho_2 F_{2\to 2} & \dots & -\rho_2 F_{2\to n} \\ \dots & \dots & \dots & \dots \\ -\rho_n F_{n\to 1} & -\rho_n F_{n\to 2} & \dots & 1-\rho_n F_{n\to n} \end{bmatrix}^{-1} \begin{bmatrix} B_{e,1} \\ B_{e,2} \\ \dots \\ B_{e,n} \end{bmatrix}$$

$O(n^3)$

Jacobi

- For all patches i (i=1...N) : $B_i^{(0)} = B_{e,i}$
- while not converged:
  - for all patches i (i=1...N)

$$B_i^{(g)} = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j^{(g-1)} F(i \to j)$$

update of 1 patch requires evaluation of N FFs

Improved Gathering

- Jacobi iteration only uses values of previous iterations to compute new values
- Gauss-Seidel iteration
  - New values used immediately
  - Slightly better convergence

Gauss-Seidel

- For all patches i (i=1...N) : $B_i^{(0)} = B_{e,i}$
- while not converged:
  - for all patches i (i=1...N)

$$B_i^{(g)} = B_{e,i} + \rho_i \sum_{j=1}^{i-1} B_j^{(g)} F(i \to j) + \rho_i \sum_{j=i}^{N} B_j^{(g-1)} F(i \to j)$$

Southwell Iteration

- “Shooting” method
- Start with initial guess for light distribution (light sources)
- Select patch and distribute its energy over all polygons

Progressive Refinement

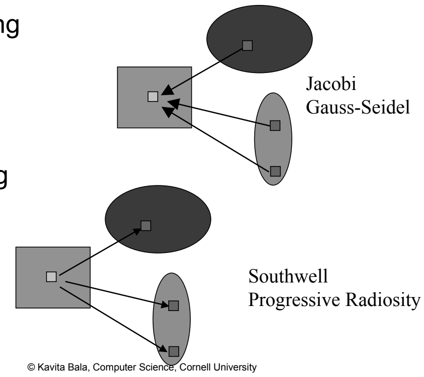
- Southwell selects shooting patches in no particular order
- Progressive refinement radiosity selects patch with largest unshot energy
- First image is generated fairly quickly!

PR + Ambient term

- PR gives an estimate for each radiosity value that is smaller than the real value
- Estimate can be improved by using ambient term
  - Add all unshot energy
  - Distribute total unshot energy equally over all patches
- Solution has improved energy distribution

Gathering vs. Shooting

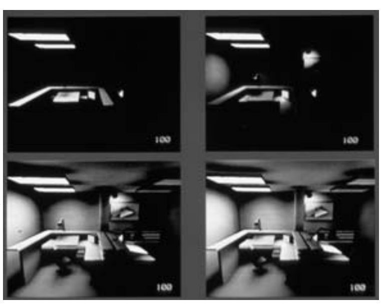
- Gathering
- Shooting



Comparison

Gauss-Seidel

Southwell



Southwell + sorting

Southwell +sorting +ambient

Radiosity Algorithms

- Object (scene) based
- Assumptions
  - Polygons
  - Diffuse BRDFs
  - Diffuse light sources
  - Static scenes
  - "Constant polygon" assumption does not capture high frequency illumination (e.g. shadow cast by a fence)

Radiosity

- Does not handle non-diffuse surfaces
- "Constant polygon" assumption does not capture high frequency illumination (e.g. shadow cast by a fence)
- Non-polygonal objects are a problem

Improvements

- Hierarchical Radiosity
- Discontinuity-Driven Radiosity

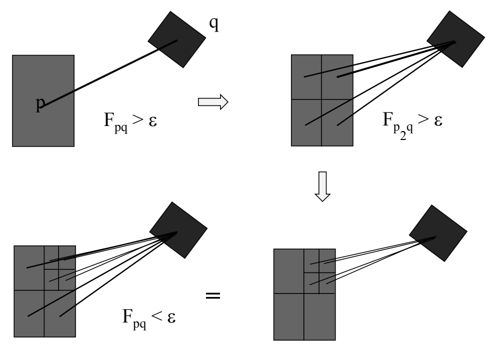
Hierarchical Radiosity

- Iterative techniques are $O(n^2)$/iteration
- Insight:
  - Radiosity is similar to N-body problem
  - Form Factor F proportional to $1/r^2$
  - Use N-body algorithms to get O(n)

HR: Main idea

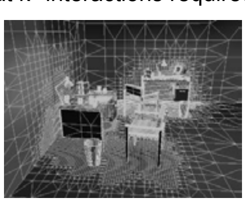
- Hierarchical refinement of patches
- Interactions at different levels of hierarchy
- Higher level interactions replace whole blocks of element-element interactions

Refining a Link



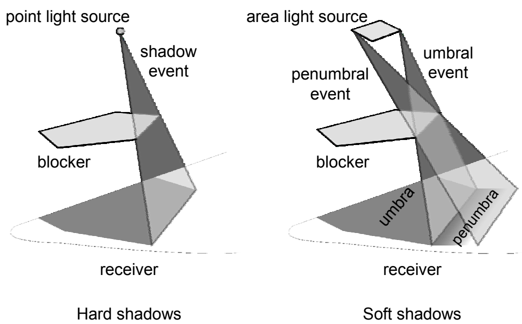
Algorithm

- Top-down
- Each node connected to constant # of nodes
- Number of interactions $O(k^2 + n)$
  - Note that k^2 interactions required

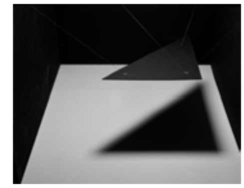


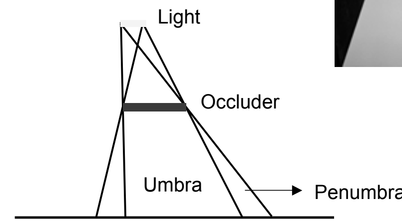
Discontinuity Meshing

Shadows: Hard and Soft



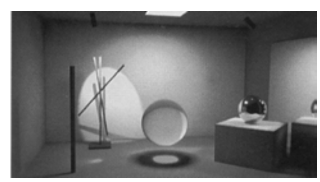
Discontinuity Meshing





Hierarchical Radiosity with Discontinuity Meshing

Complex BRDFs



Summary

- Discrete form of rendering equation
- Form Factor computations
- Different ways of solving the linear system (Jacobi, Gauss Seidel, Southwell)
- HR, Importance, Discontinuities

CS 665 Radiometry and the Rendering Equation

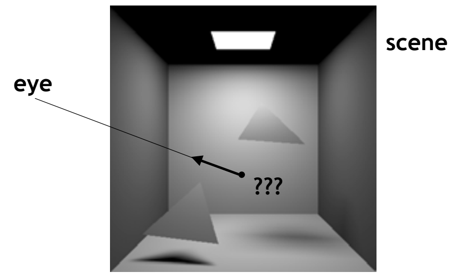
Chapter 2 in Advanced GI

What is this course about

- What does image generation mean?
  - Physics of light
- How to generate images?
  - Global illumination algorithms
- How do we this efficiently?
  - Alternative representations, data structures, image-based rendering etc.

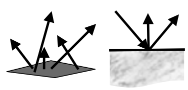
RTGI, Program of Computer Graphics

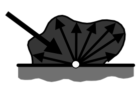
Motivation



What is the behavior of light?

- Physics of light
- Radiometry
- Material properties
- **Rendering Equation**

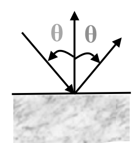




Brief history of Optics: 350 B.C.

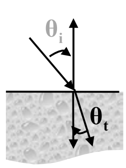
- Greek philosophers (350 B.C.)
  - Pythagoras, Plato, Aristotle
  - Light emanated from the eye
- By 300 B.C.,
  - Understood rectilinear propagation of light
  - Euclid described law of reflection

reflection



Brief history of Optics: Dark Ages

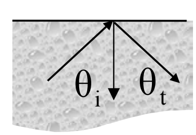
- Ptolemy (130 A.D.)
  - Refraction



- Not much in Dark Ages except for Alhazen (elaborated law of reflection)
  - Angles of incidence and reflection in the same plane as normal N

Brief history of Optics: 17th century

- 17th century: telescopes, microscopes
- Kepler (1611)
  - Total internal reflection

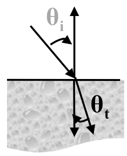


Brief history of Optics: 17th century

- Snell (1621): Law of refraction

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i}$$

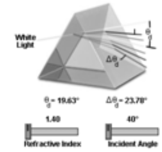
$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$



- Descartes published the sine form of law of refraction
- Fermat (1657)
  - Law of refraction from *principle of least time*

Brief history of Optics: 17th century

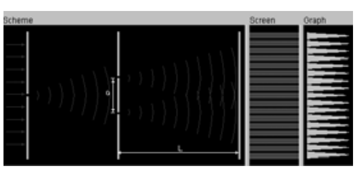
- Newton (1642-1727)
  - Dispersion
    - Light splits into component colors
  - Corpuscular/emission theory



- Huygens (1629-1695)
  - Developed wave theory of light
  - Discovered polarization
- Two conflicting theories: wave vs. emission

Brief history of Optics (contd.)

- Young (1801)
  - Principle of interference
  - Double-slit experiment



- Fresnel
  - 1816: Diffraction and interference
  - 1821: Fresnel equations for reflection and refraction

Brief history of Optics (contd.)

- Maxwell (1831-1879)
  - Electricity and magnetism
  - Maxwell's equations

$\nabla \cdot \mathbf{E} = 0$

$\nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$

$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$

  - Theoretical validation of measured speed of light
- Light is electromagnetic radiation

Brief history of Optics (contd.)

- Hertz (1887)
  - Discovered the photoelectric effect

    The process whereby electrons are liberated from materials under the action of radiant energy

- Einstein (1905)
  - Explained photoelectric effect
  - Light is a stream of quantized energy packets called quanta (photons)
  - $E = h\nu$

Brief history of Optics (contd.)

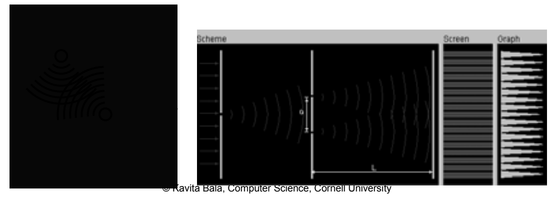
- Particle and wave theory seemingly mutually exclusive
- Quantum Mechanics
  - Bohr, Born, Heisenberg, Schrodinger, Pauli
  - Sub-microscopic phenomena
  - Unite particle and wave behavior of light
  - QED by Feynman

Models of Light

- Geometric Optics
  - Emission
  - Reflection / Refraction
  - Absorption
- Simplest model
- Size of objects > wavelength of light

Models of light

- Wave Model
  - Maxwell's Equations
  - Object size comparable to wavelength
  - Diffraction, Interference, Polarization

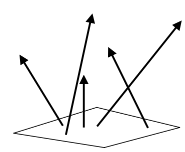


Models of light

- Quantum Model
  - Fluorescence
  - Phosphorescence
  - Relativistic effects
- Most complete model

Geometric Optics: Properties

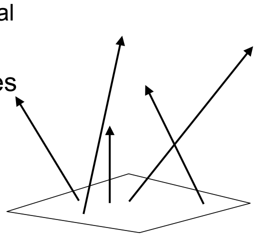
- Light travels in straight lines
- Rays do not interact with each other
- Rays have color(wavelength), intensity



Emission

- New rays due to:
  - chemical
  - electrical
  - nuclear

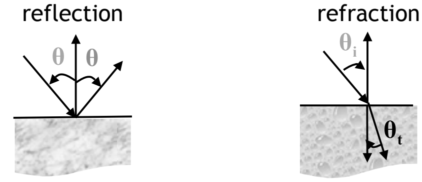
  processes



Reflections/Refractions

- Interface between 2 materials

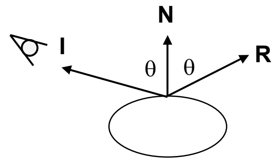
reflection refraction



- Specular reflections and refractions
  - One direction

Specular Reflections

Reflected ray R

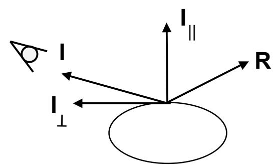


$I_{||} = (I.N)\,N$

$I_{\perp} = I - (I.N)\,N$

$R = I_{||} + (-I_{\perp})$

$\quad = 2\,(I.N)\,N - I$



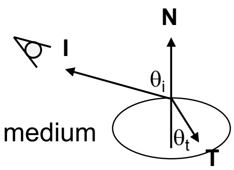
Specular Refractions

- Compute refracted ray T
- Index of refraction: $\eta = c_{vacuum} / c_{material}$
- Snell's law:

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i}$$

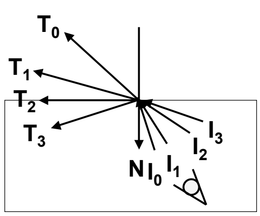
$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

- Ray from rare to dense medium



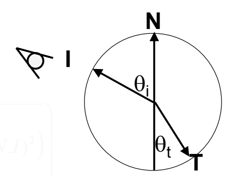
Total Internal Reflection

- Consider ray from dense to rare



$$\frac{\sin\theta_i}{1} = \frac{\eta_t}{\eta_i}$$

Transmitted Ray



$$T = -\frac{\eta_i}{\eta_t} I + N\left(\frac{\eta_i}{\eta_t}(N.I) - \sqrt{1 - \left(\frac{\eta_i}{\eta_t}\right)^2 (1 - (N.I)^2)}\right)$$

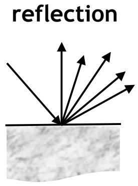
$$T = -\eta\, I + N\left(\eta \cos\theta_i - \sqrt{1 - \eta^2 \sin^2\theta_i}\right)$$

$$\cos\theta_i = (N.I)$$

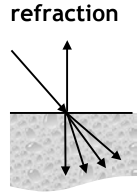
$$\eta = \frac{\eta_i}{\eta_t}$$

Realistic Reflections/Refractions

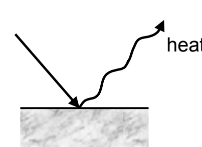
reflection



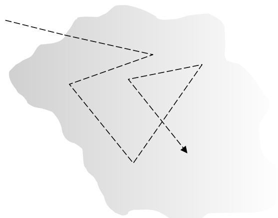
refraction



Absorption



Interaction with matter

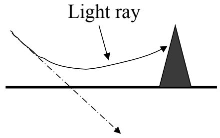


participating medium

Interaction with matter

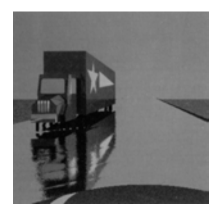
Interaction with matter

- Continuously varying refractive index



Interaction with matter

(Minnaert)



(Berger)