Lecture 3: Radiometry

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Radiosity Algorithm

• Subdivide scene in polygons
  – mesh that determines final solution
• Compute Form Factors
  – transfer of energy between polygons
• Solve linear system
  – results in power (color) per polygon
• Pick a viewpoint and display
  – loop

Form Factor

• $F_{ji} = \text{the fraction of power emitted by } j, \text{ which is received by } i$
• Area
  – if $i$ is smaller, it receives less power
• Orientation
  – if $i$ faces $j$, it receives more power
• Distance
  – if $i$ is further away, it receives less power

Form Factors - how to compute?

• Closed Form
  – Analytic
• Hemicube
• Monte-Carlo

How To Solve Linear System

• Matrix Inversion
• Gathering methods
  – Jacobi iteration
  – Gauss-Seidel
• Shooting
  – Southwell iteration
  – Improved Southwell iteration
### Matrix Inversion

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} 1 - \rho F_{i,e} & -\rho F_{i,e} & \cdots & -\rho F_{i,e} \\ -\rho F_{j,e} & 1 - \rho F_{j,e} & \cdots & -\rho F_{j,e} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho F_{N,e} & -\rho F_{N,e} & \cdots & 1 - \rho F_{N,e} \end{bmatrix} \begin{bmatrix} B_{i,1} \\ B_{i,2} \\ \vdots \\ B_{i,N} \end{bmatrix}$$

$O(n^3)$

### Jacobi

- For all patches $i$ ($i=1...N$): $B_i^{(0)} = B_{e,i}$

- while not converged:
  - for all patches $i$ ($i=1...N$)
    $$B_i^{(e)} = B_{i,e} + \rho \sum_{j=1}^{N} B_j^{(e-1)} F(i \rightarrow j)$$

- First image is generated fairly quickly!

### Improved Gathering

- Jacobi iteration only uses values of previous iterations to compute new values

- Gauss-Seidel iteration
  - New values used immediately
  - Slightly better convergence

### Gauss-Seidel

- For all patches $i$ ($i=1...N$): $B_i^{(0)} = B_{e,i}$

- while not converged:
  - for all patches $i$ ($i=1...N$)
    $$B_i^{(e)} = B_{i,e} + \rho \sum_{j=1}^{N} B_j^{(e-1)} F(i \rightarrow j) + \rho \sum_{j=1}^{N} B_j^{(e-1)} F(i \rightarrow j)$$

### Southwell Iteration

- “Shooting” method

- Start with initial guess for light distribution (light sources)

- Select patch and distribute its energy over all polygons

### Progressive Refinement

- Southwell selects shooting patches in no particular order

- Progressive refinement radiosity selects patch with largest unshot energy

- First image is generated fairly quickly!
PR + Ambient term

- PR gives an estimate for each radiosity value that is smaller than the real value

- Estimate can be improved by using ambient term
  - Add all unshot energy
  - Distribute total unshot energy equally over all patches

- Solution has improved energy distribution

Gathering vs. Shooting

- Gathering

- Shooting

Comparison

<table>
<thead>
<tr>
<th>Gauss-Seidel</th>
<th>Southwell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southwell</td>
<td>Southwell</td>
</tr>
<tr>
<td>+ sorting</td>
<td>+ sorting + ambient</td>
</tr>
</tbody>
</table>

Radiosity Algorithms

- Object (scene) based
- Assumptions
  - Polygons
  - Diffuse BRDFs
  - Diffuse light sources
  - Static scenes
  - “Constant polygon” assumption does not capture high frequency illumination (e.g. shadow cast by a fence)

Radiosity

- Does not handle non-diffuse surfaces
- “Constant polygon” assumption does not capture high frequency illumination (e.g. shadow cast by a fence)
- Non-polygonal objects are a problem
Improvments

- Hierarchical Radiosity
- Discontinuity-Driven Radiosity

Hierarchical Radiosity

- Iterative techniques are $O(n^2)$/iteration

Insight:
- Radiosity is similar to N-body problem
- Form Factor $F$ proportional to $1/r^2$
- Use N-body algorithms to get $O(n)$

HR: Main Idea

- Hierarchical refinement of patches
- Interactions at different levels of hierarchy
- Higher level interactions replace whole blocks of element-element interactions

Algorithm

- Top-down
- Each node connected to constant # of nodes
- Number of interactions $O(k^2 + n)$
  - Note that $k^2$ interactions required

Refining a Link

- $F_{pq} > \varepsilon$  $\Rightarrow$  $F_{p2q} > \varepsilon$
- $F_{pq} < \varepsilon$

Discontinuity Meshing
Shadows: Hard and Soft

- Point light source
- Area light source
- Shadow event
- Penumbra event
- Blocker:
- Receiver
- Hard shadows
- Soft shadows

Discontinuity Meshing

- Light
- Occluder
- Umbra
- Penumbra

Hierarchical Radiosity with Discontinuity Meshing

Complex BRDFs

Summary

- Discrete form of rendering equation
- Form Factor computations
- Different ways of solving the linear system (Jacobi, Gauss Seidel, Southwell)
- HR, Importance, Discontinuities
What is this course about

- What does image generation mean?
  - Physics of light
- How to generate images?
  - Global illumination algorithms
- How do we this efficiently?
  - Alternative representations, data structures, image-based rendering etc.

Motivation

- Light emanated from the eye
- Understood rectilinear propagation of light
- Euclid described law of reflection

What is the behavior of light?

- Physics of light
- Radiometry
- Material properties
- Rendering Equation

Brief history of Optics: 350 B.C.

- Greek philosophers (350 B.C.)
  - Pythagoras, Plato, Aristotle
  - Light emanated from the eye
- By 300 B.C.,
  - Understood rectilinear propagation of light
  - Euclid described law of reflection

Brief history of Optics: Dark Ages

- Ptolemy (130 A.D.)
  - Refraction
- Not much in Dark Ages except for Alhazen (elaborated law of reflection)
  - Angles of incidence and reflection in the same plane as normal N

Brief history of Optics: 17th century

- 17th century: telescopes, microscopes
- Kepler (1611)
  - Total internal reflection
Brief history of Optics: 17th century

- Snell (1621): Law of refraction
  \[ \frac{\sin \theta_i}{\sin \theta_r} = \frac{n_r}{n_i} \]
  \[ n_i \sin \theta_i = n_r \sin \theta_r \]
- Descartes published the sine form of law of refraction
- Fermat (1657)
  - Law of refraction from principle of least time

Brief history of Optics (contd.)

- Young (1801)
  - Principle of interference
  - Double-slit experiment
- Fresnel
  - 1816: Diffraction and interference
  - 1821: Fresnel equations for reflection and refraction

Brief history of Optics (contd.)

- Maxwell (1831-1879)
  - Electricity and magnetism
  - Maxwell’s equations
  \[ \nabla \cdot \mathbf{E} = 0 \]
  \[ \nabla \cdot \mathbf{B} = 0 \]
  \[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
  \[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]
  - Theoretical validation of measured speed of light
- Hertz (1887)
  - Discovered the photoelectric effect
  - The process whereby electrons are liberated from materials under the action of radiant energy
- Einstein (1905)
  - Explained photoelectric effect
  - Light is a stream of quantized energy packets called quanta (photons)
  - \( E = h \nu \)

Brief history of Optics (contd.)

- Newton (1642-1727)
  - Dispersion
    - Light splits into component colors
  - Corpuscular/emission theory
- Huygens (1629-1695)
  - Developed wave theory of light
  - Discovered polarization
  - Two conflicting theories: wave vs. emission
- Quantum Mechanics
  - Bohr, Born, Heisenberg, Schrodinger, Pauli
  - Sub-microscopic phenomena
  - Unite particle and wave behavior of light
  - QED by Feynman
Models of Light

• Geometric Optics
  – Emission
  – Reflection / Refraction
  – Absorption

• Simplest model

• Size of objects > wavelength of light

Models of light

• Wave Model
  – Maxwell’s Equations
  – Object size comparable to wavelength
  – Diffraction, Interference, Polarization

Models of light

• Quantum Model
  – Fluorescence
  – Phosphorescence
  – Relativistic effects

• Most complete model

Geometric Optics: Properties

• Light travels in straight lines

• Rays do not interact with each other

• Rays have color(wavelength), intensity

Emission

• New rays due to:
  – chemical
  – electrical
  – nuclear processes

Reflections/Refractions

• Interface between 2 materials

• Specular reflections and refractions
  – One direction
Specular Reflections

Reflected ray \( R \)

\[
\begin{align*}
I_\parallel &= (I \cdot N) N \\
I_\perp &= I - (I \cdot N) N \\
R &= I_\parallel + (-I_\perp) \\
&= 2 (I \cdot N) N - I
\end{align*}
\]

Specular Refractions

- Compute refracted ray \( T \)
- Index of refraction:
- Snell’s law:

\[
\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}
\]

- Ray from rare to dense medium

Total Internal Reflection

- Consider ray from dense to rare

Transmitted Ray

\[
T = \frac{n_i}{n_t} \sin \theta_i \left[ 1 - \left( \frac{n_t}{n_i} \right)^2 (\cos \theta_i - \cos \theta_t) \right]
\]

Realistic Reflections/Refractions

Absorption

heat
Interaction with matter

- Continuously varying refractive index

Light ray

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