CS 664
Visual Motion

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Visual Motion

- Over sequence of images can determine which pixels move where
- Differs from motion in the world
  - Camera motion
    - Pan, tilt, zoom
  - Motion parallax
    - Information about depth from camera motion
  - Scene motion
    - Reveals independent objects and behaviors
  - Un-detectable motion
    - No/low intensity variation

Motion Analysis in Video

- Video insertion
  - Compute motion in one image sequence
  - Use to transform frames of another sequence and superimpose
  - Today used to insert signs and markings into sporting events
- Panoramic mosaics with variations in depth

Estimating Visual Motion

- Historically two different approaches
  – Direct methods, based on local image derivatives at each pixel
  – Feature based methods, sparse correspondence
- We will focus on direct methods
  – Used most in practice
  – Recover image motion from spatio-temporal variations in brightness
  – Dense estimates but can be sensitive to variations in appearance

Direct Motion Estimation Methods

- Based on the following assumptions
  - Every pixel in image I goes to some location in subsequent image J
  - Overall brightness of images I,J does not change (much)
- Called brightness constancy equation

$$I(x,y) \approx J(x+u(x,y),\ y+v(x,y))$$

I

1	2	3	4
5	6	7	8
9	10	11	12
15	16	13	14

J

5	6	7	8
1	2	3	4
12	11	10	9
13	14	15	16

u

0	0	0	0
0	0	0	0
3	1	-1	-3
2	2	-2	-2

v

1	1	1	1
-1	-1	-1	-1
0	0	0	0
0	0	0	0

Using Brightness Constancy

- Minimization formulation
  - Seek (u(x,y),v(x,y)) minimizing error
    $(I(x,y)-J(x+u(x,y),y+v(x,y))^2$
  - Not practical to search explicitly!
- Linearization
  - Relate motion to image derivatives
    - Gradient constraint
  - Assuming small u,v (on order of a pixel)
  - First order term of Taylor series expansion of brightness constancy

Gradient Constraint

- One-dimensional example – linearization
  - Estimate displacement d using derivative
    - Two functions f(x) and g(x)=f(x-d)
  - Taylor series expansion

$$f(x-d) = f(x) - d\, f'(x) + E$$

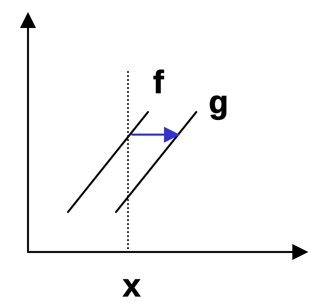
    - Where f' denotes derivative
  - Now write difference as

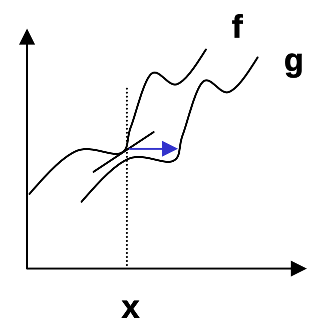
$$f(x)-g(x) = d\, f'(x) + E$$

  - Neglecting higher order terms

$$d = (f(x)-g(x))/f'(x)$$

  - Note only for small d





Gradient Constraint (or Optical Flow Constraint)

- Same approach extends naturally to 2D

$$I(x,y) \approx J(x+\mathbf{u},y+\mathbf{v}),\ \mathbf{u}=u(x,y),\ \mathbf{v}=v(x,y)$$

  - Assume time-varying image intensity well approximated by first order Taylor series

$$J(x+\mathbf{u},y+\mathbf{v}) \approx I(x,y)+I_{\mathbf{x}}(x,y)\cdot\mathbf{u}+I_{\mathbf{y}}(x,y)\cdot\mathbf{v}+I_{\mathbf{t}}$$

  - Substituting

$$I_{\mathbf{x}}(x,y)\cdot\mathbf{u}+I_{\mathbf{y}}(x,y)\cdot\mathbf{v} \approx -I_{\mathbf{t}}$$

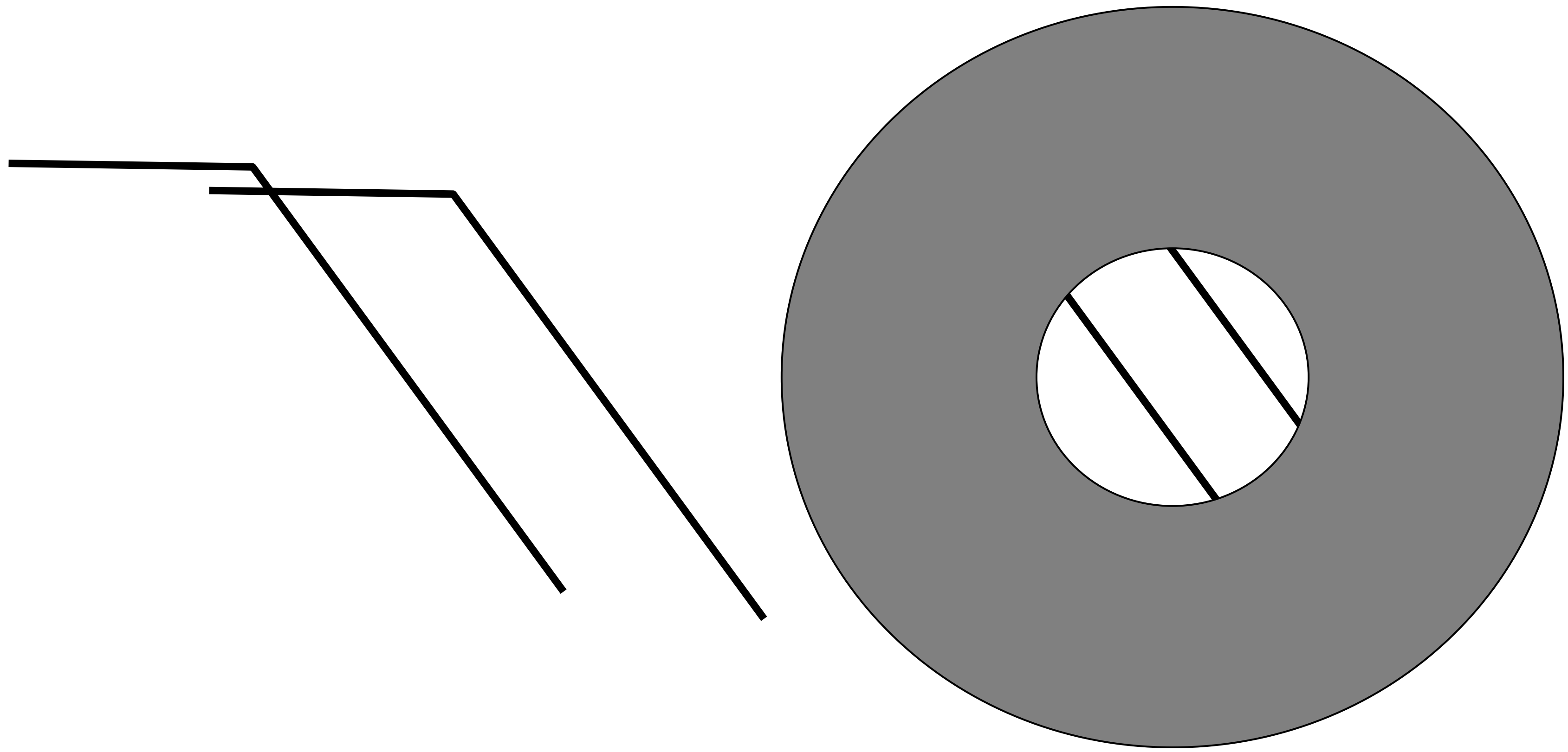
  - Using gradient notation

$$\nabla I\cdot(\mathbf{u},\mathbf{v}) \approx -I_{\mathbf{t}}$$

  - Linear constraint on motion (**u**,**v**) at each pixel
  - Can only estimate motion in gradient direction

Aperture Problem (Normal Flow)

- Can only measure motion in direction normal to edge (along gradient)

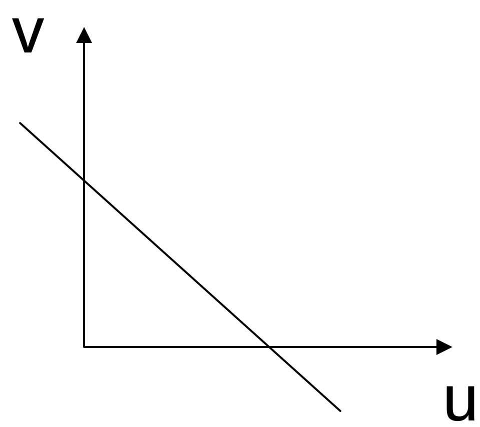


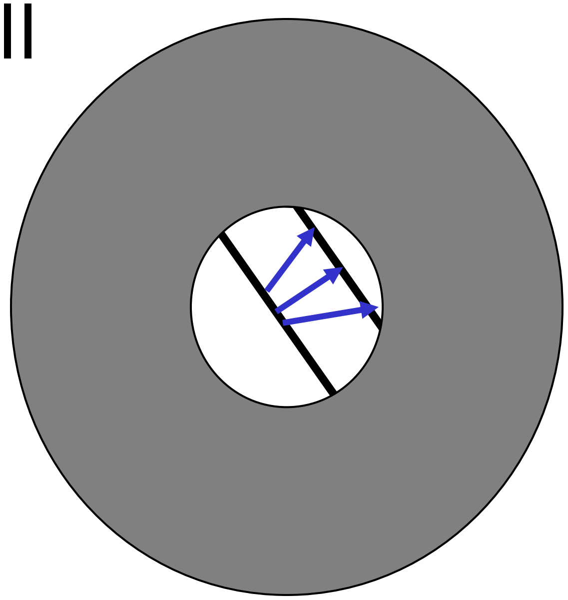
Aperture Problem (Normal Flow)

- Gradient constraint defines line in (u,v) space

$$\nabla I \cdot (\mathbf{u}, \mathbf{v}) \approx -I_{\mathbf{t}}$$

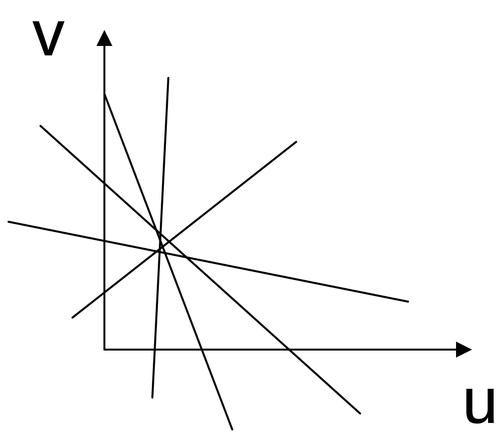
- Methods based solely on per pixel estimates don’t work well

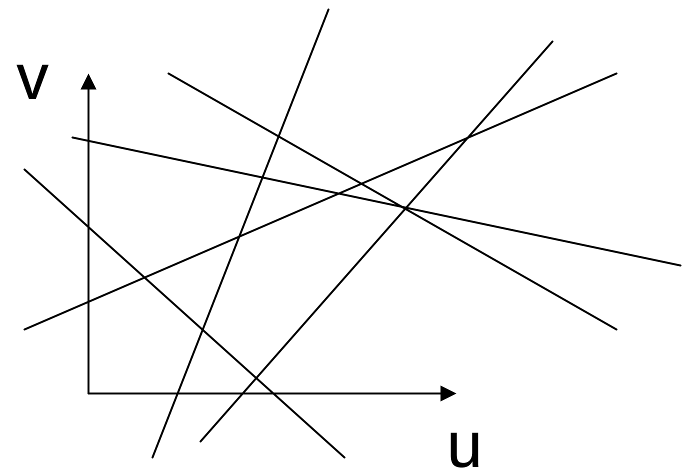




Combining Local Constraints

- Each pixel defines linear constraint on possible (u,v) displacement
  – For set of pixels with same displacement combine constraints to get estimate
  – For pixels with different displacements, somehow identify that is case





Patch Translation [Lucas-Kanade]

Assume a single velocity for all pixels within an image patch

$$E(u,v) = \sum_{x,y\in\Omega} \left(I_x(x,y)u + I_y(x,y)v + I_t \right)^2$$

Minimizing

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} \sum I_x I_t \\ \sum I_y I_t \end{pmatrix}$$

$$\left(\sum \nabla I \nabla I^T \right) \vec{U} = -\sum \nabla I I_t$$

LHS: sum of the 2x2 outer product of the gradient vector

The Aperture Problem

Let $M = \sum (\nabla I)(\nabla I)^T$ and $b = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$

- Algorithm: At each pixel compute u by solving $Mu = b$
- M is singular if all gradient vectors point in the same direction
  - e.g., along an edge
  - of course, trivially singular if the summation is over a single pixel or there is no texture
  - i.e., only *normal flow* is available (aperture problem)
- Corners and textured areas are OK

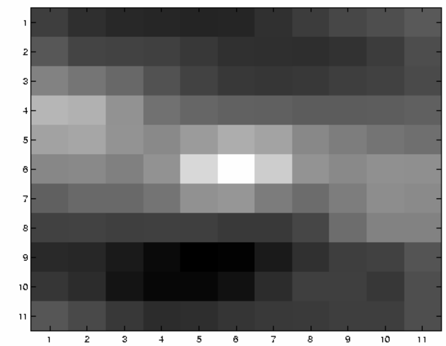
Least Squares Solution

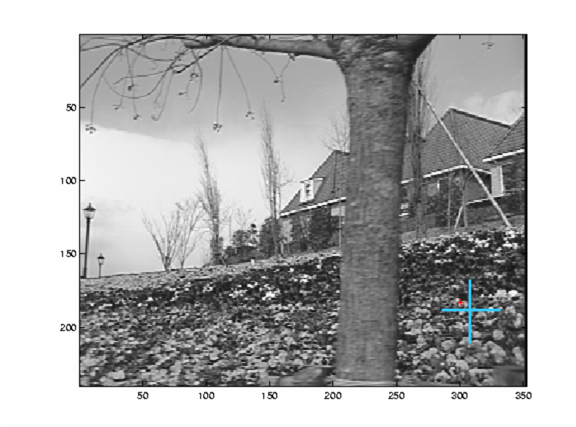
- u minimizing $Mu=b$
- Compute $(M^TM)^{-1} M^Tb$
  - Method of normal equations, can derive from setting partial derivatives to zero
  - Closed form for 2x2

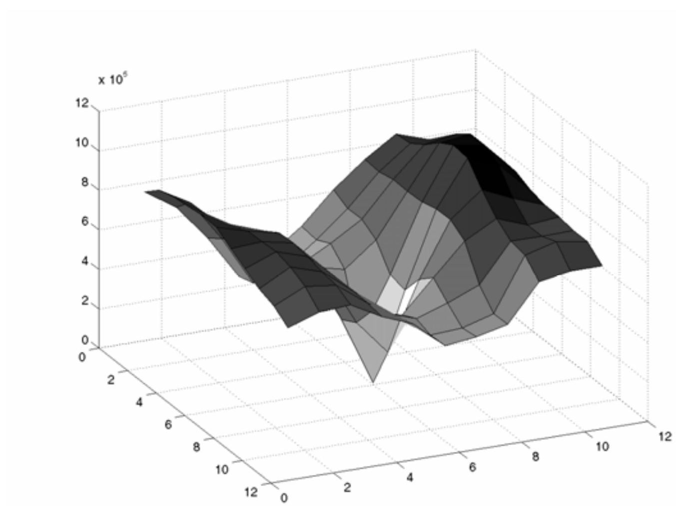
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^{-1} = 1/(ad-bc)\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Where det(A)=ad-bc not (near) zero

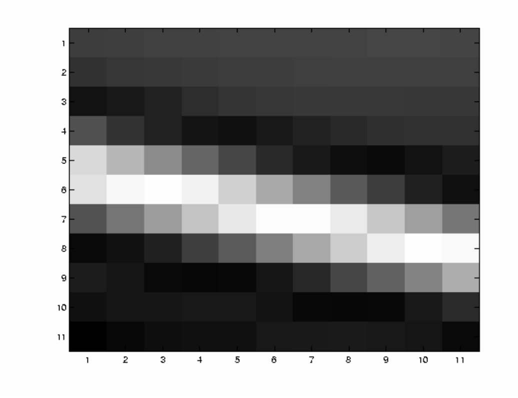
SSD Surface in Textured Area

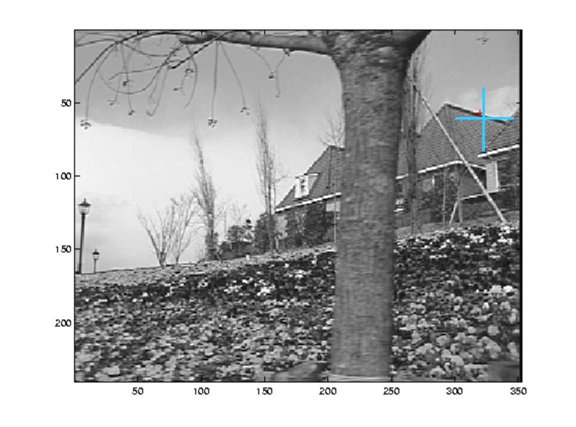


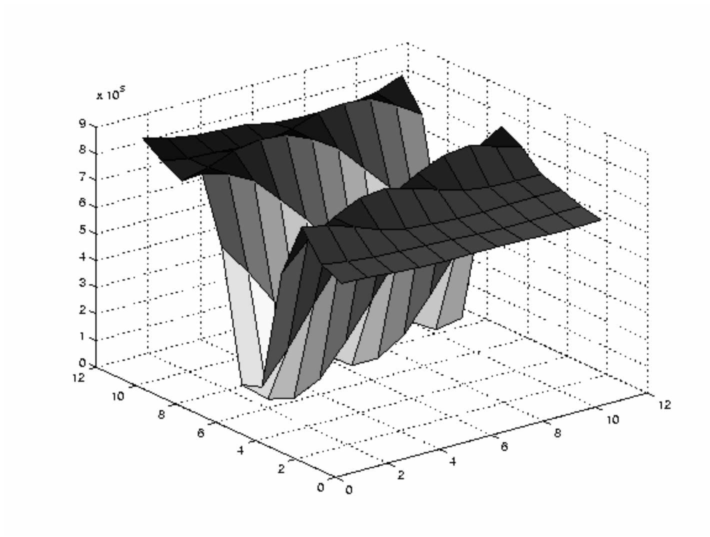




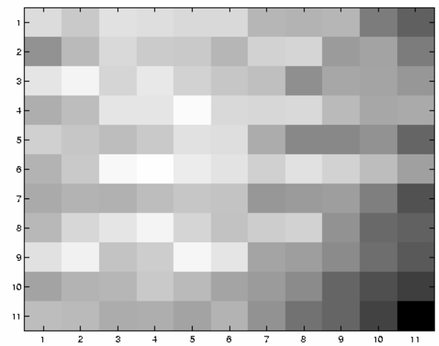
SSD Surface at an Edge

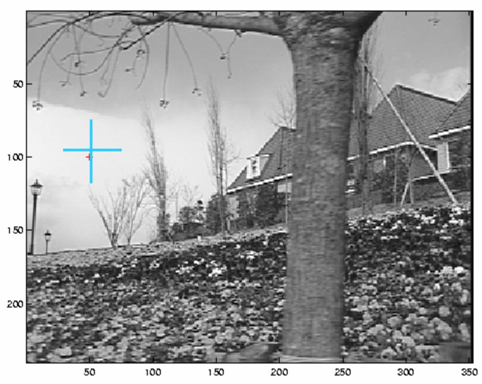


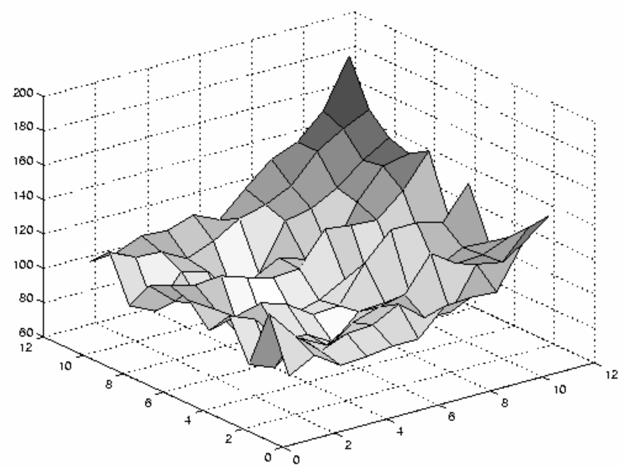




SSD in Homogeneous Area







Translational Motion

- Can estimate small translation over local patch around each pixel
  - Fast using box sums
  - Note relation to corner detection
  - Poor estimate if matrix nearly singular
  - Also poor if patch contains more than one underlying motion
- Improvements
  - Multiple motions – robust statistical techniques
  - Larger translations – pyramid methods

Multiple Motions

- Robust statistical techniques for finding predominant motion in a region
- Consider approach of iteratively reweighted least squares (IRLS)
  - As illustration of robust methods
- Generalize minimization problem to

$$\min_{\mathbf{u}} \|W(Mu - b)\|$$

  - Weight matrix W is diagonal
  - Lessen importance of pixels that don't match
  - Iterate to find "good" weights
  - Note in unweighted case W is identity matrix

Finding Predominant Motion

- Minimization generalizes in obvious way

$$u^* = (M^T W^2 M)^{-1} M^T W^2 b$$

- Determining good weights to use
  - Start by computing least squares solution, u^0
  - Iteratively compute better solutions
    - Compute error for each pixel based on previous solution u^{k-1} and use that to set weight per pixel
  - Depends on initial solution being good enough to allow “bad pixels” to have largest error
    - Have to measure error based on image intensity matches, it’s the only thing we can measure

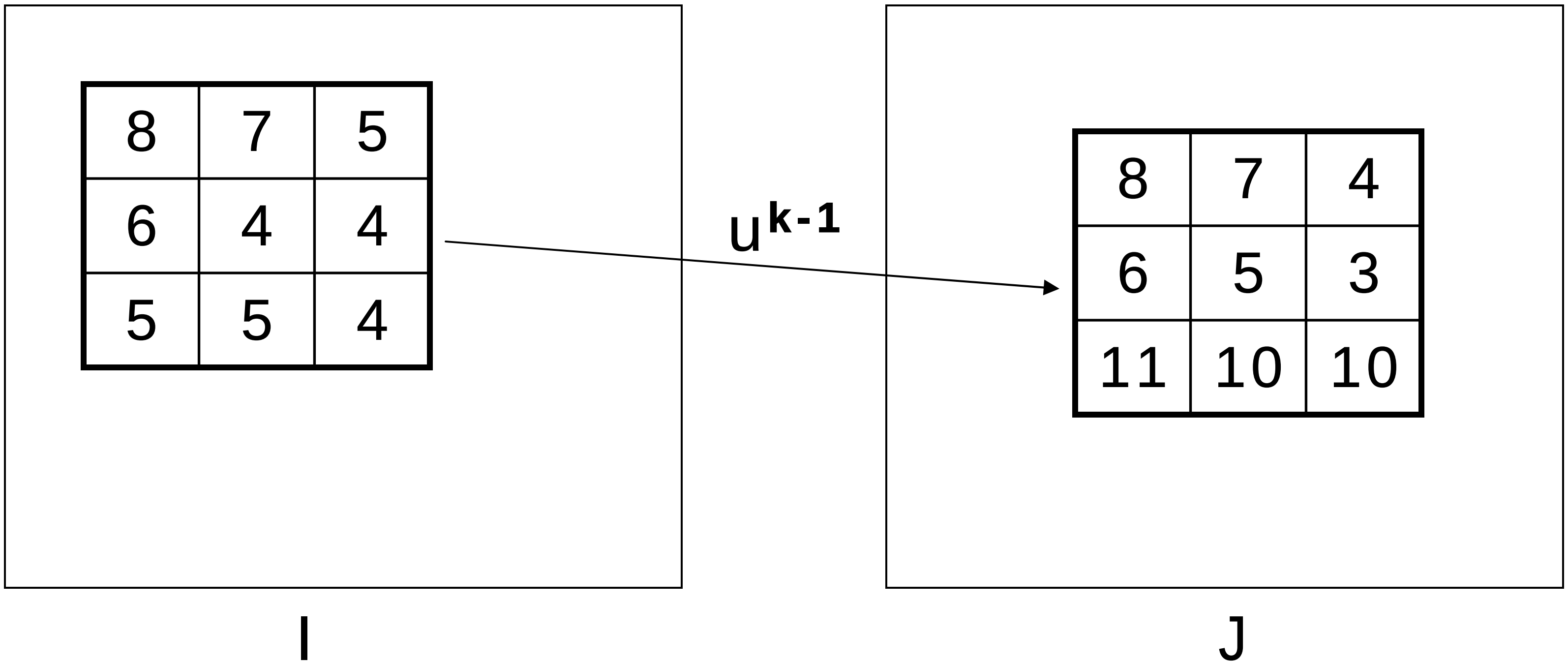
Updating Weights

- To solve for u^k given u^{k-1}
  - Create weights $W^k = \mathrm{diag}(w_1^k \ldots w_n^k)$ where

$$w_i^k = \begin{cases} 1 \text{ if } r_i^{k-1} \leq c \\ c/r_i^{k-1} \text{ otherwise} \end{cases}$$

  - Where r_i^{k-1} is measure of error at i-th pixel with motion estimate from iteration k-1
    - Compare i-th pixel value to matching pixel of other image (using u^{k-1} for correspondence)
  - And c is set based on robust measure of good versus bad data, such as median
    - Common value is 1/.6745 median(r_i^{k-1})

Weights Example



r_i^{k-1}: 0,0,1,0,1,1,6,5,6

median = 1

$c \approx 1.48$

w_i^{k}: 1,1,1,1,1,1,.24,.29,.24

Global Motion Estimation

- Estimate motion vectors that are parameterized over some region
  - Each vector fits some low-order model of how vectors change
- Affine motion model is commonly used

  $u(x,y) = a_1+a_2x+a_3y$

  $v(x,y) = a_4 + a_5x + a_6y$
- Substituting into gradient constraint eqn.

  $I_x(a_1+a_2x+a_3y) + I_y(a_4 + a_5x + a_6y) \approx -I_t$
  - Each pixel provides a linear constraint in six unknowns

Affine Transformations

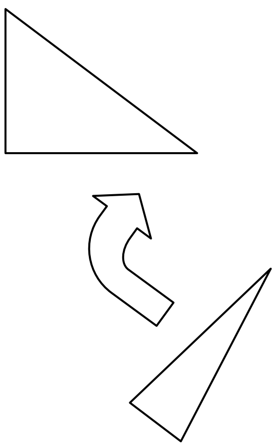
- Consider points (x,y) in plane rather than vectors for the moment
  - Linear transformation and translation

    $x' = a_1 + a_2 x + a_3 y$

    $y' = a_4 + a_5 x + a_6 y$

  - In matrix form $A(z) = Lz + b$

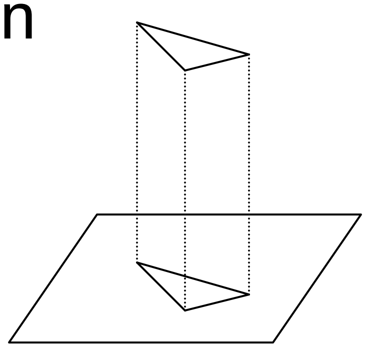
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a_2 & a_3 \\ a_5 & a_6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a_1 \\ a_4 \end{pmatrix}$$



  - Maps any triangle to any triangle
    - Defined by three corresponding pairs of points

Why Affine Transformations

- Simple (and often inaccurate) model of projection
  - Point (x,y,z) in space maps to (x,y) in image
  - Orthographic or parallel projection



- Somewhat reasonable model for telephoto lens
- Yields affine transformation of plane for viewing “flat objects”
  - 3D rotation, translation followed by orthographic projection and scaling

Affine Motion Estimation

- Minimization problem become that of estimating the parameters a_1, ... a_6
  - Rather than just two parameters u,v
- Still (over-constrained) linear system but in more unknowns
  - Again use least squares to solve
- Separable into two independent 3 variable problems
  - a_1, a_2, a_3 reflect only u-component of motion
  - a_4, a_5, a_6 reflect only v-component of motion

Affine Motion Equations

- Again compute $(D^T D)^{-1} D^T t$
  - Or (re)weighted version for IRLS
- Now two 3x3 problems, one for I_x and one for I_y, as opposed to single 2x2 problem
- Problem for I_x and u motion (I_y analogous)
  - T remains same, D changes

$$D = \begin{pmatrix} I_{x1}\, x_1 & I_{x1}\, y_1 & I_{x1} \\ \vdots & \vdots & \vdots \\ I_{xn}\, x_n & I_{xn}\, y_n & I_{xn} \end{pmatrix}$$

Multiple (Layered) Motions

- Combining global parametric motion estimation with robust estimation
  - Calculate predominant parameterized motion over entire image (e.g., affine)
  - Corresponds to largest planar surface in scene under orthographic projection
    - If doesn’t occupy majority of pixels robust estimator will probably fail to recover its motion
  - Outlier pixels (low weights in IRLS) are not part of this surface
    - Recursively try estimating their motion
    - If no good estimate, then remain outliers

Other Global Motion Models

- The affine model is simple but not that accurate in some imaging situations
  - For instance "pinhole" rather than "parallel" camera model for closer objects
  - Non-planar surfaces
  - Explicit modeling of motion parallax
- Projective planar case

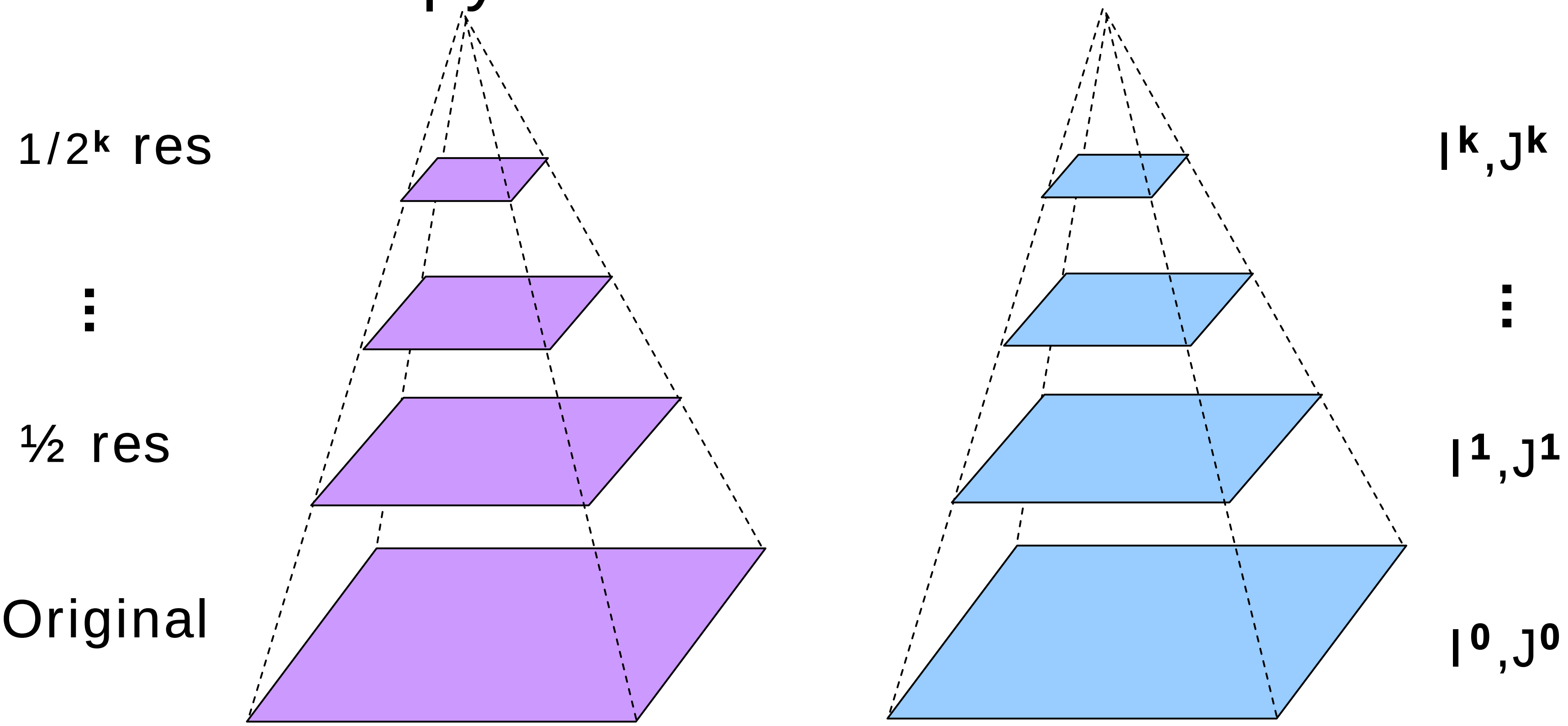
  $x' = (h_1+h_2x+h_3y)/(h_7+h_8x+h_9y)$

  $y' = (h_4+h_5x+h_6y)/(h_7+h_8x+h_9y)$

  and $u=x'-x$, $v=y'-y$
- 3D models such as residual planar parallax

Coarse to Fine Motion Estimation

- Estimate residual motion at each level of Gaussian pyramid



Pyramid of image I

Pyramid of image J

Coarse to Fine Estimation

- Compute M^k, estimate of motion at level k
  - Can be local motion estimate (u^k, v^k)
    - Vector field with motion of patch at each pixel
  - Can be global motion estimate
    - Parametric model (e.g., affine) of dominant motion for entire image
  - Choose max k such that motion about one pixel
- Apply M^k at level k-1 and estimate remaining motion at that level, iterate
  - Local estimates: shift I^k by $2(u^k, v^k)$
  - Global estimates: apply inverse transform to J^{k-1}

Global Motion Coarse to Fine

- Compute transformation Tk mapping pixels of I^k to J^k
- Warp image J^{k-1} using T^k
  - Apply inverse of $T^{\mathbf{k}}$
  - Double resolution of $T^{\mathbf{k}}$ (translations double)
- Compute transformation T^{k-1} mapping pixels of I^k to <u>warped</u> J^{k-1}
  - Estimate of "residual" motion at this level
  - Total estimate of motion at this level is composition of $T^{\mathbf{k-1}}$ and resolution doubled $T^{\mathbf{k}}$
    - In case of translation just add them

Affine Mosaic Example

- Coarse-to-fine affine motion
  - Pan tilt camera sweeping repeatedly over scene
- Moving objects removed from background
  - Outliers in motion estimate, use other scans

SSD

- An alternative to gradient based methods is template matching
  - Treat a rectangle around each pixel as a “template” to find best match in other image
  - Search over possible translations minimizing some error criterion (or maximizing quality)
  - Generally use sum squared difference (SSD)

    $\Sigma\,\Sigma\,(I(x,y)-J(x+u,y+v))^2$
  - Sometimes compute cross correlation
  - Compute over local neighborhood