CS 664 Lecture 3
Hausdorff and Chamfer Matching

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Distance Transforms in Matching

- Chamfer measure – asymmetric
  - Sum of distance transform values
    - "Probe" DT at locations specified by model and sum resulting values
- Hausdorff distance (and generalizations)
  - Max-min distance which can be computed efficiently using distance transform
  - Generalization to quantile of distance transform values more useful in practice
- Iterated closest point (ICP) like methods
  - Fitzgibbons

Hausdorff Distance

- Classical definition
  - Directed distance (not symmetric)
    - $h(A,B) = \max_{\mathbf{a} \in \mathbf{A}} \min_{\mathbf{b} \in \mathbf{B}} \| a-b \|$
  - Distance (symmetry)
    - $H(A,B) = \max(h(A,B), h(B,A))$
- Minimization term is simply a distance transform of B
  - $h(A,B) = \max_{\mathbf{a} \in \mathbf{A}} D_{\mathbf{B}}(a)$
  - Maximize over selected values of DT
- Classical distance not robust, single "bad match" dominates value

Distance Transform Definition

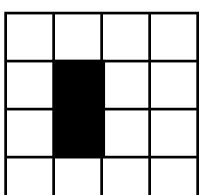
- Set of points, P, some distance $\|\bullet\|$

$$D_P(x) = \min_{y \in P} \|x - y\|$$

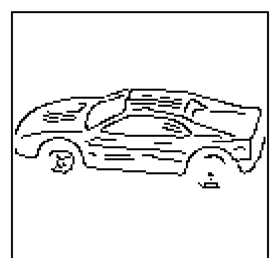
  - For each location x distance to nearest y in P
  - Think of as cones rooted at each point of P
- Commonly computed on a grid Γ using

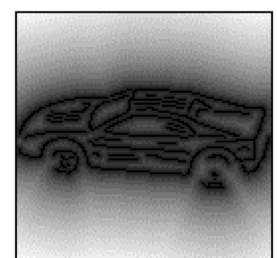
$$D_P(x) = \min_{y \in \Gamma} \left(\|x - y\| + 1_P(y) \right)$$

  - Where $1_P(y) = 0$ when $y \in P$, ∞ otherwise



2	1	2	3
1	0	1	2
1	0	1	2
2	1	2	3



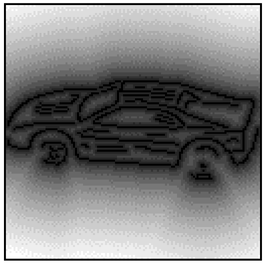


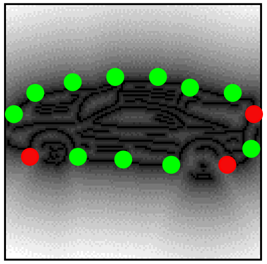
Hausdorff Matching

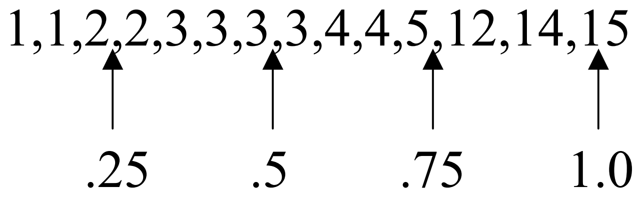
- Best match
  - Minimum fractional Hausdorff distance over given space of transformations
- Good matches
  - Above some fraction (rank) and/or below some distance
- Each point in (quantized) transformation space defines a distance
  - Search over transformation space
    - Efficient branch-and-bound “pruning” to skip transformations that cannot be good

Hausdorff Matching

- Partial (or fractional) Hausdorff distance to address robustness to outliers
  - Rank rather than maximum
    - $h_{\mathbf{k}}(A,B) = \text{kth}_{\mathbf{a}\in\mathbf{A}} \min_{\mathbf{b}\in\mathbf{B}} \| a\text{-}b \| = \text{kth}_{\mathbf{a}\in\mathbf{A}} D_{\mathbf{B}}(a)$
  - K-th largest value of $D_{\mathbf{B}}$ at locations given by A
  - Often specify as fraction f rather than rank
    - 0.5, median of distances; 0.75, 75th percentile





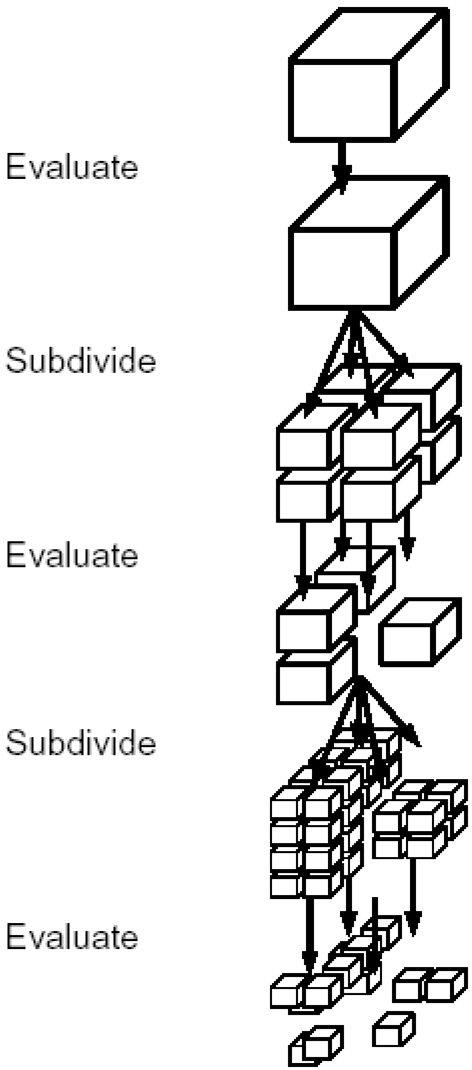


Fast Hausdorff Search

- Branch and bound hierarchical search of transformation space
- Consider 2D transformation space of translation in x and y
  - (Fractional) Hausdorff distance cannot change faster than linearly with translation
    - Similar constraints for other transformations
  - Quad-tree decomposition, compute distance for transform at center of each cell
    - If larger than cell half-width, rule out cell
    - Otherwise subdivide cell and consider children

Branch and Bound Illustration

- Guaranteed (or admissible) search heuristic
  - Bound on how good answer could be in unexplored region
    - Cannot miss an answer
  - In worst case won't rule anything out
- In practice rule out vast majority of transformations
  - Can use even simpler tests than computing distance at cell center

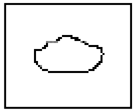


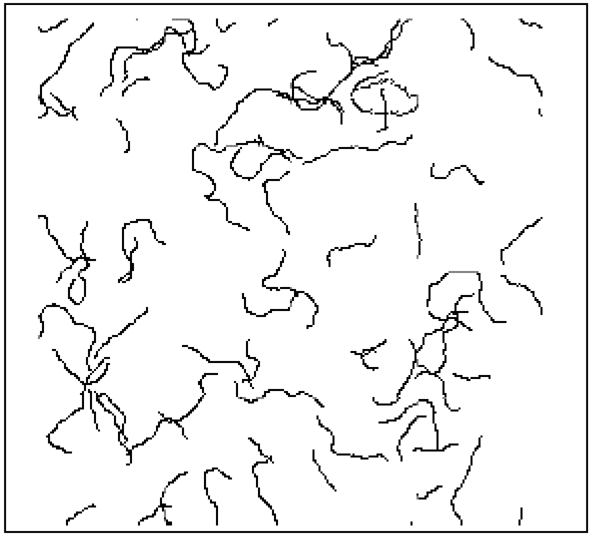
DT Based Matching Measures

- Fractional Hausdorff distance
  - Kth largest value selected from DT
- Chamfer
  - Sum of values selected from DT
    - Suffers from same robustness problems as classical Hausdorff distance
    - Max intuitively worse but sum also bad
  - Robust variants
    - Trimmed: sum the K smallest distances (same as Hausdorff but sum rather than largest of K)
    - Truncated: truncate individual distances before summing

Comparing DT Based Measures

- Monte Carlo experiments with known object location and synthetic clutter
  - Matching edge locations
- Varying percent clutter
  - Probability of edge pixel 2.5-15%
- Varying occlusion
  - Single missing interval, 10-25% of boundary
- Search over location, scale, orientation

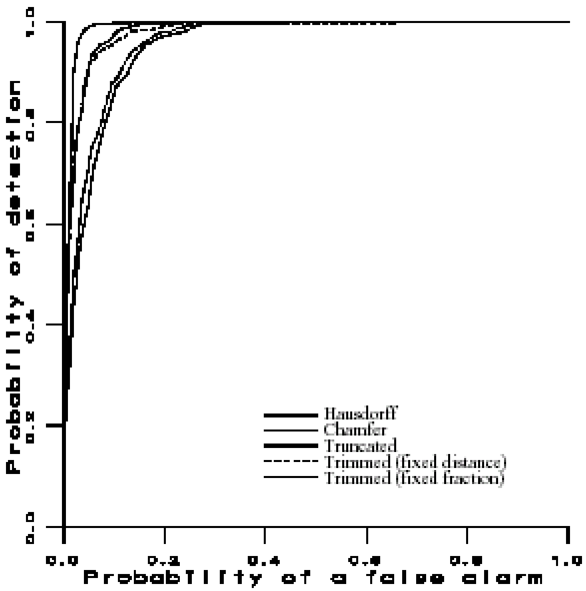


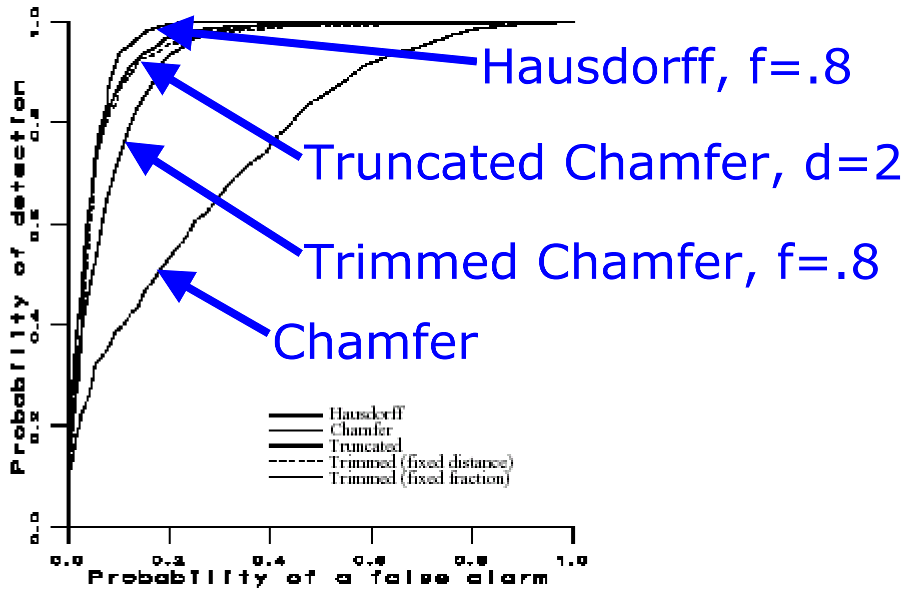


5% Clutter Image

ROC Curves

- Probability of false alarm vs. detection
  - 10% and 15% occlusion with 5% clutter
  - Chamfer is lowest, Hausdorff (f=.8) is highest
  - Chamfer truncated distance better than trimmed



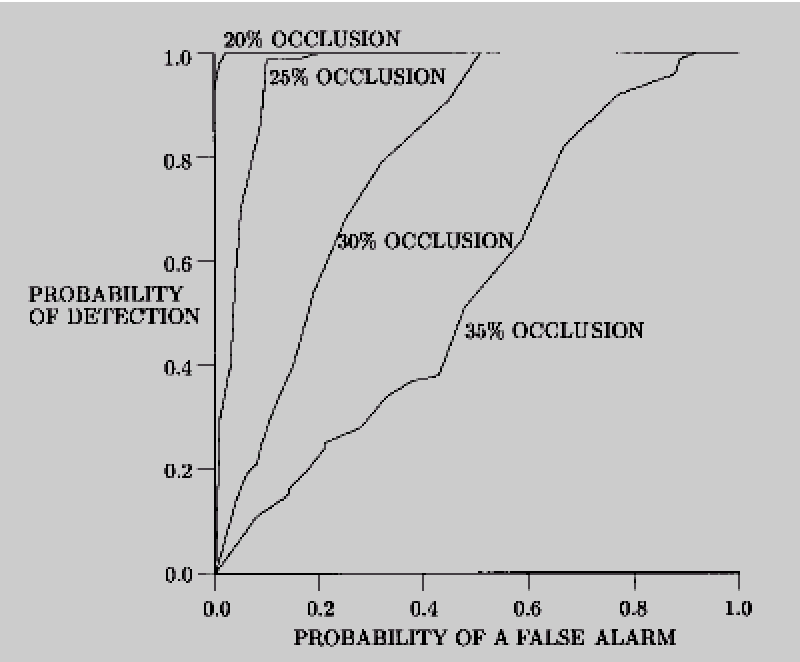


Edge Orientation Information

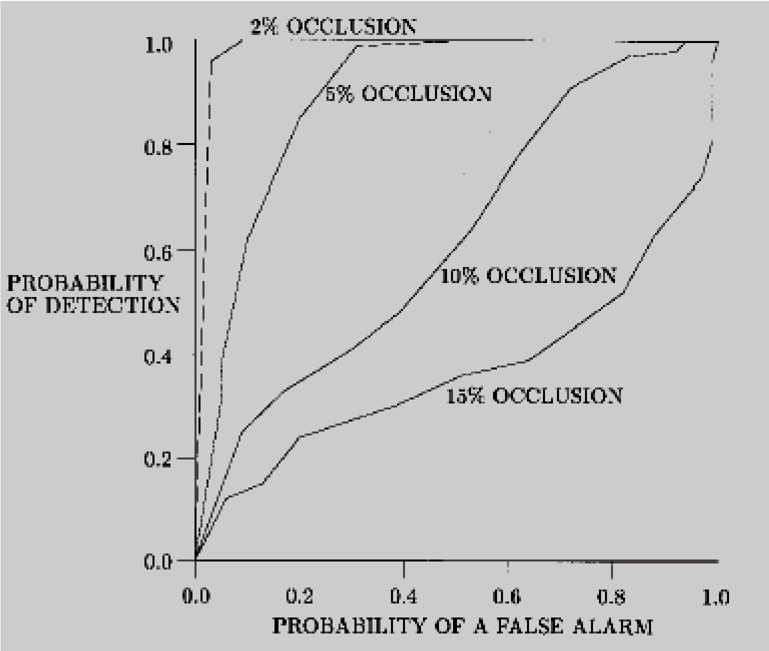
- Match edge orientation as well as location
  - Edge normals or gradient direction
- Increases detection performance and speeds up matching
  - Better able to discriminate object from clutter
  - Better able to eliminate cells in branch and bound search
- Distance in 3D feature space $[p_{\mathbf{x}}, p_{\mathbf{y}}, \alpha p_{\mathbf{o}}]$
  - α weights orientation versus location
  - $\mathrm{kth}_{\mathbf{a} \in \mathbf{A}} \min_{\mathbf{b} \in \mathbf{B}} \| a\text{-}b \| = \mathrm{kth}_{\mathbf{a} \in \mathbf{A}} D_{\mathbf{B}}(a)$

ROC's for Oriented Edge Pixels

- Vast improvement for moderate clutter
  - Images with 5% randomly generated contours
  - Good for 20-25% occlusion rather than 2-5%



Oriented Edges



Location Only

Observations on DT Based Matching

- Fast compared to explicitly considering pairs of model and data features
  - Hierarchical search over transformation space
- Important to use robust distance
  - Straight Chamfer very sensitive to outliers
    - Truncated DT can be computed fast
- No reason to use approximate DT
  - Fast exact method for L_2^2 or truncated L_2^2
- For edge features use orientation too
  - Comparing normals or using multiple edge maps