CS 664 Slides #10
Structure From Motion

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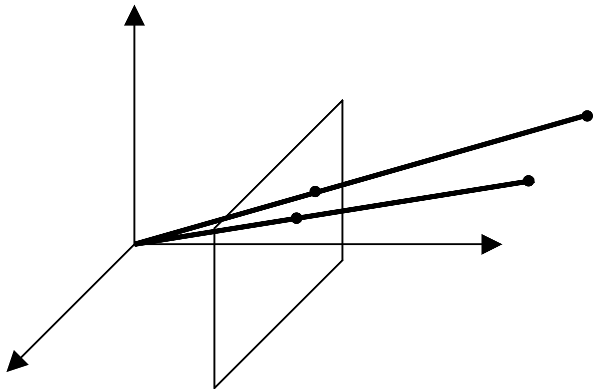
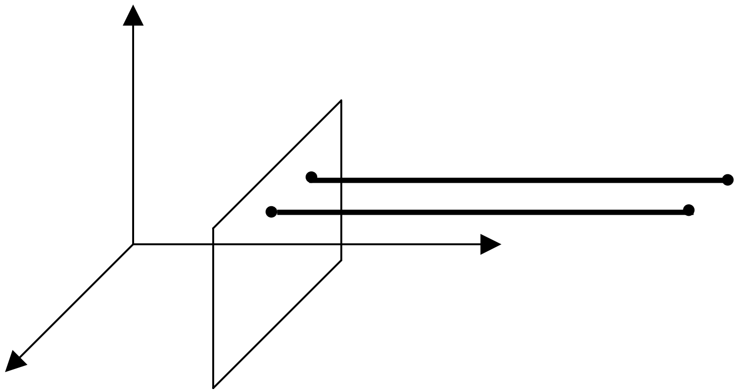
Structure From Motion

- Recover 3D coordinates from set of 2D views
  - Rigid body motion
  - Known correspondence of points in views
  - Various camera models
- Consider representative case of
  - Parallel (orthographic) projection
  - All points visible in all views
  - Un-calibrated camera
  - No outliers (least squares ok)

Parallel Projection

- Point (X,Y,Z) in space projects to (X,Y) in image plane
  - Contrast with (fX/Z,fY/Z) in pinhole model
  - Light rays all parallel rather than through principal point
    - Similar when points at same depth, narrow FOV





Recovering 3D Structure

- With enough corresponding points and views can determine 3D locations
  - Redundant information
    - Each view changes only viewing parameters and not point locations
      - 3P unknowns for P points and kF unknowns for F views
- Minimum sufficient correspondences
  - Orthographic projection, three views of four points
  - Central (pinhole) projection, two views of eight points

Sensitive to Measurement Noise

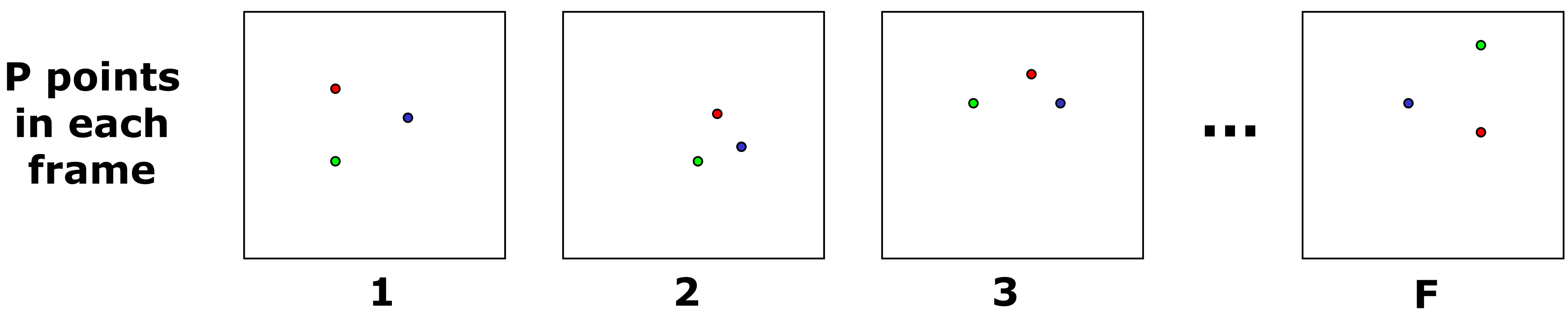
- Solutions based on a small number of points are not stable
  - Errors of the magnitude found in most images yield substantial differences in recovered 3D values
- Method that works in practice called factorization
  - Works on sequence of several frames
  - With correspondences of points
  - Consider case of factorization for orthographic projection, no outliers, can be extended

Input: Sequence of Tracked Points

- Point coordinates

$$w'_{fp}=(u'_{fp},v'_{fp})$$

  - Where f denotes frame index and p denotes point index
  - Points tracked over frames
    - E.g., use corner trackers discussed previously



Centroid Normalized Coordinates

- From observed coordinates $w'_{fp}=(u'_{fp},v'_{fp})$

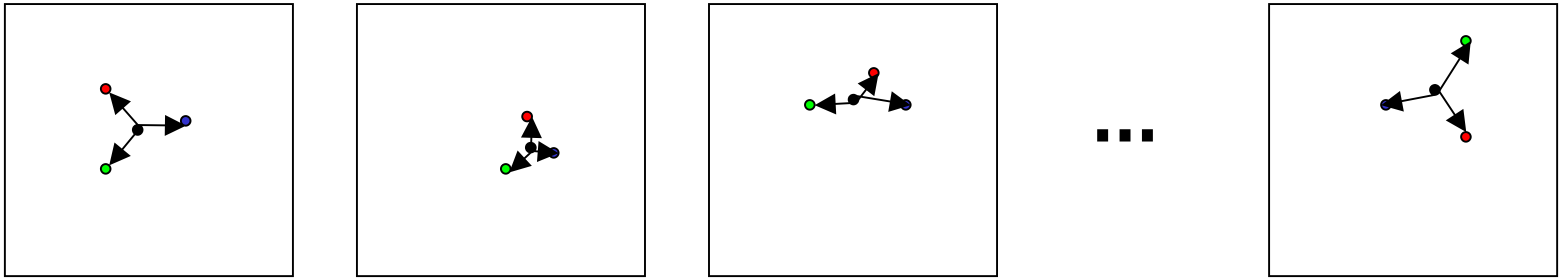
$$w_{fp}=(u'_{fp} - \bar{u}_{fp},\ v'_{fp} - \bar{v}_{fp})$$

  – Where

$$\bar{u}_{\mathbf{fp}} = (1/P)\ \textstyle\sum_{\mathbf{p}} u'_{\mathbf{fp}}$$

and Centroids

$$\bar{v}_{\mathbf{fp}} = (1/P)\ \textstyle\sum_{\mathbf{p}} v'_{\mathbf{fp}}$$



Normalization

- Goal of separating out effects of camera translation from those of rotation
- Subtract out centroid to remove translation effects
  - Assume all points belong to object and present at all frames
  - Centroid preserved under projection
- Left to recover 3D coordinates (shape) of P points from F camera orientations

Measurement Matrix

- 2FxP – 2 rows per frame, one col per point
- In absence of senor noise this matrix is highly rank deficient
  - Under orthographic projection rank 3 or less

$$W = \begin{bmatrix} u_{11} & \cdots & u_{1P} \\ \vdots & & \vdots \\ u_{F1} & \cdots & u_{FP} \\ v_{11} & \cdots & v_{1P} \\ \vdots & & \vdots \\ v_{F1} & \cdots & v_{FP} \end{bmatrix}$$

Structure of W

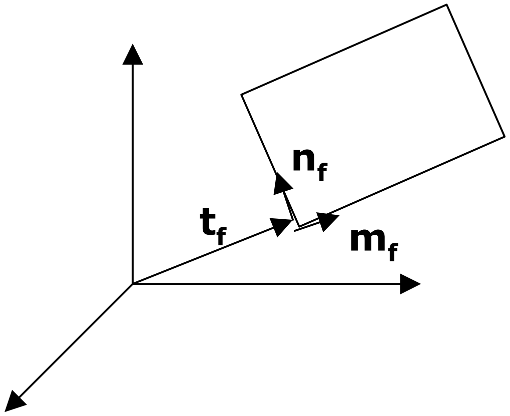
- World point $s_p' = (x_p', y_p', z_p')$ projects to image points

$$u'_{fp} = m_f^T (s_p' - t_f)$$

$$v'_{fp} = n_f^T (s_p' - t_f)$$

  – Where $\mathbf{m_f}$, $\mathbf{n_f}$ are unit vectors defining orientation of image plane in world

  – And $\mathbf{t_f}$ is vector from world origin to image plane origin



Structure of W (Cont'd)

- Can rewrite in centroid normalized coordinates
  - Since centroid preserved under projection
  - Projection of centroid is centroid of projection

$$u_{fp} = m_f^T s_p$$
$$v_{fp} = n_f^T s_p$$

  - Where

$$s_{\mathbf{p}} = s_{\mathbf{p}}' - \bar{s}$$

and

$$\bar{s} = (1/P) \sum_{\mathbf{p}} s'_{\mathbf{p}}$$

W Factors Into Simple Product

- W=MS where
  - M is 2Fx3 matrix of camera locations
  - S is 3xP matrix of points in world
  - Product is 2Fx3 matrix W
    - Clearly rank at most 3

$$M = \begin{bmatrix} m_1^T \\ \vdots \\ m_F^T \\ n_1^T \\ \vdots \\ n_F^T \end{bmatrix} \qquad S = \begin{bmatrix} s_1 & \cdots & s_P \end{bmatrix}$$

Factoring W

- Don't know M,S only measurements W
- When noise or errors in measurements seek least squares approximation
  - Note l.s. assumes no outliers (bad data)

$$\text{argmin}_{M,S} \| W\text{-}MS \|^2$$

- The best M,S of this form can be found using the SVD of W

  $W=U\Sigma V$

  Σ' contains only three largest singular values

  $M^*=U\ \Sigma'^{½}$

  $S^*=\ \Sigma'^{½}V$

Factorization Not Unique

- Any linear transformation of M,S possible

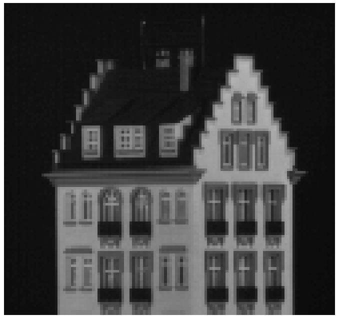
  $W=MS=M(LL^{-1})S=(ML)(L^{-1}S)$
- Often referred to as "affine shape"
  - Preserves parallelism/coplanarity
- Still haven't used a constraint on the form of M
  - Describes camera plane orientation at each frame

    m_i, n_i all unit vectors

    $m_i n_i = 0$

$$\begin{bmatrix} m_1^T \\ \vdots \\ m_F^T \\ n_1^T \\ \vdots \\ n_F^T \end{bmatrix}$$

Factorization Results



1

40

60

80

120

150