Correlation and Epipolar Geometry

CS664 Machine Vision Lecture #12

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I. Administrative

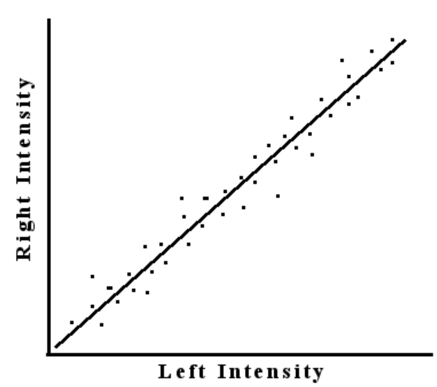
- Assignment 2 posted on web, due 10/30/01
- Quiz in lecture, 10/23/01
- Individual presentations will take place some Wednesday in November

II. Matching Under Gain and Bias

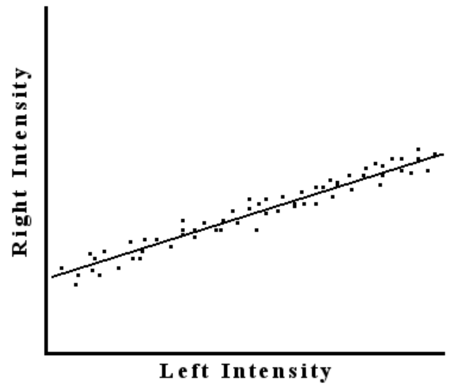
Goal: find disparity information in images where gain and bias is present
Gain – Multiplicative transformation
Bias – Additive transformation

Possible solution: add new gain and bias labels to disparity labels and solve
Problem: Quickly becomes intractable

Better solution: Correlation of paired data
Simplest Example: no gain, no bias, no nearby discontinuities, some Gaussian noise:



More complicated example: Allow gain and bias. Best-fit will still be a line, this time with varied slope and non-zero y-intercept:



If there is no correspondence between images, the graph will be random garbage.
If the slope of the line is negative, the second image would be a photographic negative.

Pearson's Correlation Coefficient

+1 = r = -1

r > 0 represents positive slope, r < 0 represents negative slope, and r = 0 means random garbage (no correlation).

Covarience:

$$co(x,y) = \sum_i (x_i - \bar{x})(y_i - \bar{y})$$

$$\frac{co(x,y)}{\sigma_x \sigma_y} = r$$

r is called the Normalized Coefficient. It is much like computing L2 distance. It is non-robust on edges and it doesn't tolerate outliers.

III. Non-Parametric Statistics

Distribution-free, takes advantage of ordering information

Given a set (x_i, y_i) in any non-Gaussian, "God Knows What" distribution, replacing the pixel intensities with their intensity rank creates a known distribution. Gain and bias have no effect upon this order.

$$(x_i, y_i) \Rightarrow (\tilde{x}_i, \tilde{y}_i), \quad i = 1..n$$

The ranking reduces the effect of outliers. It is not a great method for low-texture areas (few measures are).

Spearman Correlation - Use Pearson's Correlation but use intensity ranks rather than actual intensities. This actually calculates:

$$\Sigma(\tilde{x}_i - \tilde{y}_i)^2$$

This is a non-robust measure, but that is acceptable because there are no significant outliers in the ranks. It gives a perfect score to any monotonic function.

Problem: slow. Can't use dynamic programming tricks.
Fix: **Rank Transform –** count the number of pixels that are darker in a given window. Do this for all pixels, then use correlation. This can be done fast.

Kendall's t - points (a, b) and (c, d) lie on monotonically increasing function. They are concordant if:

$$(a < c ? b < d) V (a > c ? b > d)$$

Kendall's t counts the number of concordant pairs. More pairs indicates more correlation.
Problem: same as before, slow.

Census Transform – world's fastest stereo computations, can be done in real time using specialized hardware.

Idea: for each pixel, create a bit-string of comparisons to other pixels. 0 indicates darker, 1 otherwise. Then compare strings using Hamming Distance (XOR and count).

Sparse vs. Dense Output – only compute disparities for a few important pixels. Other values can be determined from these.

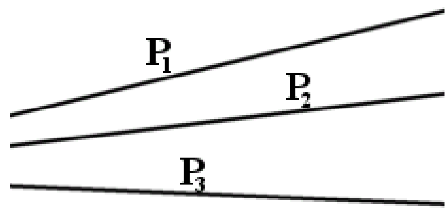
Why do this? May be faster and more accurate since looking at fewer pixels. It measures correspondence only at "interesting" points: edges, corners, features.

Problem: Can't use dynamic programming. Also generally less accurate due to less information, less robust. It can work well in certain situations.

IV. Epipolar Geometry

If we had perfect information with a certain number of points, how many points would we need?

Consider two cameras C_1 and C_2, and three pixels P_1, P_2, and P_3. The image of a ray from C_1 to P_1 will be the search space for P_1 in C_2



View from C_2

The lines converge to hit camera 1. The point where the lines meet in C_2 is called the **Epipole** – Image of C_1's center in C_2. If both cameras are looking straight ahead the lines are parallel and the Epipole is at infinity.

A scene point and C_1 and C_2 create an **Epipolar Plane**

Epipolar Line – Intersection of epipolar plane and imaging surface. All epipolar planes intersect to create a pencil of planes.

In stereo vision, epipolar lines are horizontal. Epopolar geometry can be represented in matrix form – the Fundamental matrix and the Essential matrix.

(To be continued in lecture 13)

Relation of Epipolar Geometry to Sparse Correspondence – When trying to compute motion, you can take a few points and solve for the Fundamental and Essential matrices. Then you have a sense of the relative motion of the camera.