A Signal-Processing Framework for Inverse Rendering

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CS 6630 Realistic Image Synthesis
Inverse Rendering

Photographs → Inverse Rendering Algorithm

Geometric model → BRDF

Lighting
Why inverse rendering

Precision measurement

Data extraction

Image-based bidirectional reflectance distribution function measurement


APPLIED OPTICS / Vol. 39, No. 16 / 1 June 2000

Face Recognition Using 3-D Models: Pose and Illumination

Novel face recognition algorithms, based on three-dimensional information, promise to improve automated face recognition by dealing with different viewing and lighting conditions.

By Sami Romdhani, Jeffrey Ho, Member IEEE, Thomas Vetter, Member IEEE, and David J. Kriegman
Signal-Processing Framework

• How much information can I extract?
• The problem is well- or ill-posed?
• What is the best way to express the model?

Before only “handwaving” explanation
Spherical Harmonics

$L=0$

$L=1$

$L=2$

$L=3$

$m=-3$  $m=-2$  $m=-1$  $m=0$  $m=1$  $m=2$  $m=3$
Spherical Harmonics

Analog to Fourier base for angles

\[ Y_{l}^{m}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l-m)!}{(l+m)!}} P_{l}^{m} \cos \theta e^{im\phi} \]

Coefficients obtained via projection

\[ C_{l}^{m} = \int \int \int f(\theta, \phi) Y_{l}^{m}(\theta, \phi)^{*} d\theta d\phi \]

Orthonormality

\[ \int Y_{l}^{m}(\theta, \phi) Y_{l'}^{m'}(\theta, \phi)^{*} d\theta d\phi = \delta_{m,m'} \delta_{l,l'} \]
Inverse rendering

- Known geometry
- Fixed ‘far’ light
- Reflection

Looks like convolution
(and convolution simple in Fourier’s space)
Inverse Rendering

- Plugging in SH
  \[
  B(\ldots, L_{lq,s}^{m,p,r}, Y_{lq,s}^{m} R_{\ldots}, Y_{p}^{l}, Y_{s}^{r})
  \]

- Rotation in SH
  \[
  Y_{lq,s}^{m} R_{\ldots}^{l} \rightarrow D_{lq,s}^{m,m'}^{l} m Y_{lq,s}^{m'}
  \]

- Symmetry in BRDF
  \[
  p, r \quad q, s \quad p \quad q, s
  \]
Inverse Rendering

• Reflection expanded in SH base

\[ B_{p,q}^{l,m} \cdot C_{p,q}^{l,m} \quad : \quad l \cdot L_m^l_{p,q} \cdot C_{p,q}^{l,m} \]

Normalization
Lightings
BRDF
Reflection (measured)
Mirror BRDF

\[ \hat{\rho}_{lpq} = (-1)^q \delta_{lp} \quad (1) \]

- The factor \((-1)^q\) is due to the change of \(\pi\) in azimuthal angle upon reflection.
- As is usual for delta functions, there is no dropoff with \(\ell\) ("frequency"). This makes the inverse lighting problem well-conditioned.
Single directional source

\[ L_{\ell m} = \delta_{m0} Y^*_{\ell0} (0) = \delta_{m0} \Lambda^{-1}_l \]  \hspace{1cm} (2)

- We define the light to be in the +z direction—the “north pole”. Spherical harmonics have the property that they have the value 1 (before normalization) at this point if \( m = 0 \), and the value 0 otherwise. Again the inverse lighting problem is well-conditioned.
Lambertian BRDF

\[ \hat{\rho}_\ell = 2\pi \int_0^{\frac{\pi}{2}} \cos \theta'_i Y_{\ell 0} (\theta'_i) \sin \theta'_i d\theta'_i \]  

- Since there is no dependence on outgoing angle, \( p \) and \( q \) don't matter.
- This is zero for all odd values of \( \ell \), except \( \ell = 1 \) (due to the clamping at the horizon).
- The even terms fall off as \( \ell^{-\frac{5}{2}} \). > 99% of the power is contained in the \( \ell \leq 2 \) terms (9 spherical harmonics). This means an approximate characterization easy, but the inverse lighting problem poorly conditioned.
Lambertian BRDF

Figure 4: Left: Successive approximations to the clamped cosine function by adding more spherical harmonic terms. For $l = 2$, we get a very good approximation. Right: The solid line is a plot of spherical harmonic coefficients $A_l = \Lambda_l \hat{\rho}_l$. For $l > 1$, odd terms vanish, and even terms decay rapidly.
Phong BRDF

\[
\hat{\rho} = \frac{s + 1}{2\pi} (\vec{R} \cdot \vec{L})^s \\
\Lambda_\ell \hat{\rho}_\ell \approx \exp\left(-\frac{\ell^2}{2s}\right) \quad s \gg 1, s \gg \ell
\]

- \( s \) is the specular exponent.
- This is approximately a Gaussian with width \( \sim \sqrt{s} \). Beyond this, the inverse rendering problem begins to become ill-conditioned.
- As \( s \to \infty \) we approach the mirror case.
- From the properties of convolution, we can represent the Phong BRDF by blurring the lighting and using a mirror BRDF instead.
Figure 5: Numerical plots of the Phong coefficients $\Lambda_1 \hat{p}_1$, as defined by equation 18. The solid lines are the approximations in equation 19.
Microfacet BRDF

\[ \Lambda_\ell \hat{\rho}_\ell \approx \exp \left( - (\sigma \ell)^2 \right) \]  

- \( \sigma \) is the surface roughness.
- Add a Fresnel factor for non-normal exitance.
- Also approximately Gaussian; now the width is \( \sim \sigma^{-1} \).
- The trick of blurring the lighting and using a mirror BRDF instead can work here too for \( \ell \ll \sigma^{-1} \).
Decomposition of Lights for Microfacets

\[ B = B_d + B_{s,\text{slow}} + B_{s,\text{fast}} \]  \hspace{1cm} (7)
\[ L = L_d + L_{s,\text{slow}} + L_{s,\text{fast}} \]  \hspace{1cm} (8)

- \( B_d \) is the diffuse component. This can be represented using just the first 9 spherical harmonic terms of the lighting.
- \( B_{s,\text{slow}} \) is the slow-varying lighting. Here we can blur the lighting and treat the BRDF as a mirror.
- \( B_{s,\text{fast}} \) is the fast-varying lighting. Here we treat the lighting as a delta function. The reflection is approximated as a Gaussian.
BRDF Recovery
From Complex Geometry
Questions?