Here is the H-G probability density, written as a function of μ, which is cos θ.

```
In[58]:= pdf[μ] = 1/2 (1 - g^2) / (1 + g^2 - 2 g μ)^(3/2)
```

Out[58]= $$\frac{1-g^2}{2\left(1+g^2-2 g \mu\right)^{3/2}}$$

Now we can get the cumulative distribution by integrating with respect to μ.

```
In[15]:= cdf[μ'] = Integrate[pdf[μ], {μ, -1, μ'}, Assumptions → {-1 < g < 1, -1 < μ' < 1, g μ' ≠ 0}]
```

Out[15]= $$\frac{(-1+g)\left(-1-g+\sqrt{1+g^2-2 g \mu'}\right)}{2 g \sqrt{1+g^2-2 g \mu'}}$$

The sampling procedure is to solve for cdf equal to ξ where ξ is a uniform random number between 0 and 1.

```
In[60]:= sample[ξ] = μ' /. First[Solve[cdf[μ'] == ξ, μ']]
```

Out[60]= $$\frac{-1+2 g-g^2+2 \xi-2 g \xi+2 g^2 \xi-2 g^3 \xi+2 g \xi^2+2 g^3 \xi^2}{(1-g+2 g \xi)^2}$$

This is the sampling function as I have it in my notes:

```
In[62]:= mysample[ξ] = (-((1 - g^2) / (1 - g + 2 g ξ))^2 + (1 + g^2)) / (2 g)
```

Out[62]= $$\frac{1+g^2-\frac{\left(1-g^2\right)^2}{(1-g+2 g \xi)^2}}{2 g}$$

Verify that it is the same as the one just derived.

```
In[64]:= Together[mysample[ξ] == sample[ξ]]
```

Out[64]= True

Here is a simplified version that may be better for evaluation:

```
In[84]:= Simplify[sample[ξ]]
```

Out[84]= $$\frac{-1+2 \xi+2 g^3(-1+\xi) \xi+g^2(-1+2 \xi)+2 g\left(1-\xi+\xi^2\right)}{(1+g(-1+2 \xi))^2}$$