## Consensus in Distributed Systems

Impossibility of Distributed Consensus with One Faulty Process and
Paxos Made Simple

## Paxos Made Simple



Leslie Lamport

- A group of agents want to agree on a common value
- They are cautious because they might have different opinions

■ Consequently, they don't write down a firm value until agreement

- Once written down, value can't be changed

■ We want all agents to eventually write down the same value

Lots of examples of consensus in distributed systems:
■ Primary replica for data object in Bayou
■ Distributed primary in Pond

- Chain replication
- Any time you determine a consistent commit ordering
- CAP Theorem: Consistency requires consensus
- Group of agents: proposers, acceptors, learners
- Agents have input/output registers for value
- Formal specification for protocol and communication
- Safety: no contradiction
- Termination: doesn't run forever


## Examples

■ Single leader

- Majority

Node failure can break protocol
Need a failure resilient consensus algorithm!

Asynchronous execution:

- Agents operate at arbitrary speed
- May fail by stopping and may restart later
- Messages can be delayed arbitrarily and delivered out of order
- No arbitrary failures

Want Paxos to be as general as possible, but
■ Mostly no hardware clocks, but may need these to fix some issues later.
■ Need some permanent store across fail/restart.

The criteria for safety in the Paxos algorithm are:

- Only a value that has been proposed may be chosen

■ Only a single value is chosen

- A process never learns that a value has been chosen unless it actually has been

Note the distinction between proposed and chosen.

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Note the distinction between proposed, accepted, and chosen. Rely on overlapping majority to make consistent choice.

■ To distinguish proposals, give every proposal a number.

- Numbers have a total order

■ No two proposals share a number

How can we make sure only one value is chosen?

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- If the proposer hears from a majority of acceptors, it will hear about $v$.
- If it hears about $v$, it needs to propose it to maintain P2b.

P2c: To make a proposal numbered $n$ with value $v$, a proposer must know of a majority $S$ of acceptors such that either

- No acceptor in $S$ has accepted anything numbered less than $n$
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P2c: To make a proposal numbered $n$ with value $v$, a proposer must know of a majority $S$ of acceptors such that either

■ No acceptor in $S$ will ever accept anything numbered less than $n$

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Instead of trying to predict the future, use promises.
In learning about acceptors, extract a promise that the information it learns will always be true.

A request for information about requests numbered less than $n$ is also a contract to never accept any proposals numbered less than $n$.

Now, protocol for Proposers/Acceptors is basically fixed:

- Proposer picks a proposal number $n$
- Sends prepare request asking for information about proposals less than $n$
- This is also a promise not to accept new proposals less than $n$
- If it hears back from a majority, it knows enough information to apply P2c
- If majority hasn't ever accepted proposals, can pick any value

■ If someone accepted with some value $v$, has to re-propose $v$

- The message with a proposal is an accept request

An acceptor can do anything it hasn't promised not to do

- It can ignore any request without compromising safety
- Can always respond to prepare request

■ Can respond to accept request if it hasn't promised otherwise

P2c needs to be maintained across failure/restart
Solution: Acceptor remembers highest numbered proposal it has ever accepted, and highest numbered promise it has made, on permanent storage.

- Since value chosen is consistent, learning is easy.

■ Somehow broadcast acceptances to all learners.

- Acceptors could all inform distinguished learner.
- Better: acceptors inform small set of learners.

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- $p_{2}$ sends proposal numbered $n_{2}$, rejected It starts again with prepare request numbered $n_{4}>n_{3}$
- $p_{1}$ sends proposal numbered $n_{1}$, rejected It starts again with prepare request numbered $n_{5}>n_{4}$
- $p_{2}$ sends proposal numbered $n_{4}$, rejected

It starts again with prepare request numbered $n_{6}>n_{5}$

■ Solve the competing proposer problem by having a distinguished proposer

- Single proposer always makes progress if enough components working.
- Single point of failure, so need a way of electing a new proposer


## Isn't that consensus again?

Lamport punts, says to use timeouts, or failure detectors.
There's actually a good reason for this

## Impossibility of Distributed Consensus with One Faulty Process



Michael Fisher


Nancy Lynch


Michael Paterson
"Window of Vulnerability"
■ Delay at the wrong time stalls system
■ Distinguishing between failed process and temporarily slow process is difficult

Want as general a model as possible

- Processes are state machines, possibly infinite states

■ Deterministic

- Fail by stopping

■ Can send arbitrary messages to other machines

- Messages uncorrupted

But system is asynchronous
■ Could take arbitrarily long between actions for process
■ Could take arbitrarily long to deliver message

- Only guarantee delivery if process tries to receive infinitely many times
- Processes have no physical clocks

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■ Admissible run: One fault, messages eventually delivered

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- Admissible run: One fault, messages eventually delivered
- Totally correct in spite of one fault: Protocol is partially correct, and every admissible run is deciding


## Theorem (Impossibility)

No consensus protocol is totally correct in spite of one fault

- Bivalent configuration: Different runs cause protocol to decide 0 and 1
- These are 'indecisive' configurations

■ Window of Vulnerability: bad run can be continually indecisive

## Lemma 1

There is a bivalent initial configuration

If we're at a bivalent configuration $C, e$ is an event we can apply, and $\mathscr{D}$ are the configurations reachable from $C$ doing e last...
... then $\mathscr{D}$ has a bivalent configuration.

If we're sitting at $C$, and $e$ is about to take us to a non-bivalent configuration:

- e 'decides' the protocol (though output may be delayed)
- Message or process for e gets delayed

■ Instead, we do some other events, eventually doing e
■ Now, we just did e, so we're in $\mathscr{D}$
■ Pick 'other stuff' appropriately, so that we're still undecided
That's the window of vulnerability.

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- End up having delivered message, but still indecisive
- Put $p$ at back of queue, repeat

No faulty processes, all messages delivered, but no decision

Just constructed indecisive run with no faults.
So, it isn't the faults themselves, but the protocol being correct in spite of faults, that causes indecision.

Thus: difficulty is in distinguishing fault from temporary delay.

Also, we only showed the existence of one bad run.
May be exceedingly unlikely.
Only applies to truly asynchronous systems.
Indecision could be resolved with physical clocks or failure detectors

■ Consensus is everywhere

- Paxos safe against failures, maybe even terminates (does it?)
- Everything has a window of vulnerability

■ If we strengthen our model, may not apply

The Weakest Failure Detector for Solving Consensus
Tushar Chandra, Vassos Hadzilacos and Sam Toueg

Simplest failure detector necessary:

- There is a time after which every failed process is suspected by some correct process
- There is a time after which some correct process is never suspected by any correct process

Can use, for instance, timeouts to give you this

