

May 7, 2020

$$f \sim \text{GP}(\mu, k) \quad X \Rightarrow f_X \sim N(\mu_X, K_{XX})$$

$$\text{Observations: } y = f_X + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2 I)$$

$$\max_{\sigma} \mathcal{L}(\sigma | y) = p_{\sigma}(y) = \frac{\exp(-\frac{1}{2} y^T K_{\sigma}^{-1} y)}{\sqrt{(2\pi)^d \det(K_{\sigma})}}$$

$$\log \mathcal{L} = -\frac{1}{2} \mathbf{y}^T \mathbf{K}_\sigma^{-1} \mathbf{y} - \frac{d}{2} \log(2\pi) - \frac{1}{2} \log \det(\mathbf{K}_\sigma)$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} \mathbf{y}^T \mathbf{K}_\sigma^{-1} \mathbf{y} = \mathbf{y}^T \mathbf{K}_\sigma^{-1} \underbrace{\frac{\partial \mathbf{K}_\sigma^{-1}}{\partial \sigma}}_{2\sigma \mathbf{I}} \mathbf{K}_\sigma^{-1} \mathbf{y}$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} \log \det(\mathbf{K}_\sigma) = \text{trace} \left(\mathbf{K}_\sigma^{-1} \underbrace{\frac{\partial \mathbf{K}_\sigma}{\partial \sigma}}_{2\sigma \mathbf{I}} \right)$$

- $\mathbf{K}_\sigma^{-1} \quad \mathbf{K}_\sigma \mathbf{z} = \mathbf{y} \quad O(N^3)$

- $\log \det(\mathbf{K}_\sigma) = \log \det(\mathbf{U} \mathbf{N} \mathbf{U}^T) = \log \det(\mathbf{U}^T \mathbf{U} \mathbf{N})$
 $= \log \prod_i \lambda_i = \sum_i \log(\lambda_i)$

- $\text{trace}(\mathbf{K}_\sigma^{-1} \cdot 2\sigma \mathbf{I}) \quad O(N^3)$

$x_i \in \mathbb{R}^2, \mathbb{R}^3$ (spatial data)

$$K(x_i, x_j) = \exp(-\|x_i - x_j\|_2^2)$$

$O(N)$

$O(N)$



hierarchical low rank

H-matrix

G220

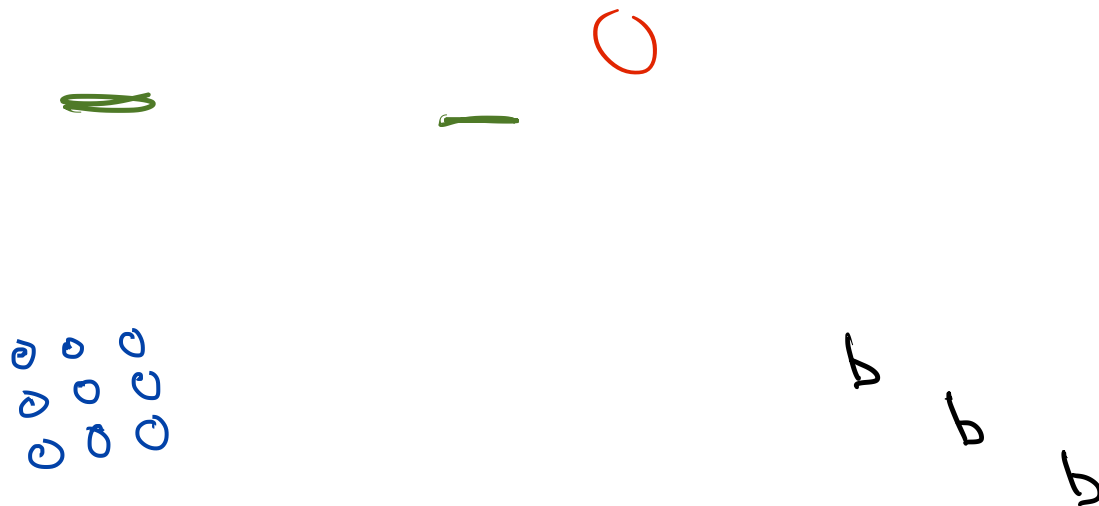
skeletonization

$Ax = b$ $\left\{ \begin{array}{l} \text{factorization } LUx = b \\ \text{iterative} \end{array} \right.$

Krylov subspace $\{b, Ab, A^2b, \dots\}$ (6210)

$$z^T \underbrace{Az}$$

Structured kernel interp (SKI)



$$(K_{XX})_{ij} = k(x_i, x_j) \approx w_i^T K_{uu} w_j$$